Use your text book to answer these questions.

Due Date: 16/07/2018

Multiple-choice questions

1 $\int_a^{a^2} 2x\ dx$ in terms of a is equal to:

A a^2

B $2a(a - 1)$

C $2a$

D a^3

E $a^2(a^2 - 1)$

2 $\int_0^2 3f(x) + 2\ dx =$

A $3\int_0^2 f(x)dx + 3x$

B $3\int_0^2 f(x)dx + x$

C $3\int_0^2 f(x)dx + 4$

D $3f'(x) + 4$

E $\int_0^2 f(x)dx + 4$

3 Given $\frac{dy}{dx} = ae^{-x} + 5$ and that when $x = 0$, $\frac{dy}{dx} = 7$ and $y = 1$, the value of y when $x = 2$ is

A $-\frac{2}{e^2} + 3$

B $-\frac{2}{e^2} + 13$

C $-\frac{3}{e^2} + 3$

D $2e^2 + 3$

E $2e^2 + 10$

4 The curve with equation $y = a \sin\left(\frac{\pi}{4}x\right) + b$ passes through the points (0, 3) and (2, 5). The area of the region enclosed between this curve, the x-axis and the lines $x = 1$ and $x = 3$ is:

A $\frac{4\sqrt{2}}{\pi} + 3$

B $\frac{4\sqrt{2}}{\pi} + 9$

C $\frac{8\sqrt{2}}{\pi} + 3$

D $\frac{8\sqrt{2}}{\pi} + 6$

E $6 - \frac{8\sqrt{2}}{\pi}$

5 Given that $\frac{d}{dx}(x\,e^{2x}) = e^{2x} + 2x\,e^{2x}$, an antiderivative of $x\,e^{2x}$ is:

A $\frac{1}{2}e^{2x} + \frac{1}{2}x^2\,e^{2x}$

B $\frac{1}{4}x^2\,e^{2x}$

C $2e^{2x} + 2x\,e^{2x}$

D $-\frac{1}{2}e^{2x} + \frac{1}{2}x\,e^{2x}$

E $-\frac{1}{4}e^{2x} + \frac{1}{2}x\,e^{2x}$

6 Given that for $f(x) = x \sin x$, $f'(x) = \sin x + x \cos x$, an antiderivative of $x \cos x$ is:

A $x \sin x$

B $x \sin x + \cos x$

C $x \sin x - \cos x$

D $2 \cos x$

E $x \sin x - \sec^2 x$

7 If $f'(x) = e^{3x}$ and $f(0) = 1$ then $f(x) =$

A $\frac{1}{3}e^{3x} - \frac{2}{3}$

B $\frac{1}{3}e^{3x} + \frac{2}{3}$

C $\frac{1}{3}e^{3x} + 1$

D $3e^{3x} - 2$

E $3e^{3x} + 1$

8 The area of the region bounded by the curve $y = \sin\frac{x}{2}$, the x-axis and the line $x = \pi$ is:

A 0

B 1

C 2

D π

E 4

9 The total area of the regions bounded by the curve $y = \cos 2x$, the lines $x = 0$ and $x = \pi$, and the x-axis is:

A 0

B 1

C 2

D $\sqrt{3}$

E $\frac{1}{\sqrt{2}}$

10 The area of the region enclosed by the curve $y = -e^{3x} + 3$, the line $x = -3$ and the x-axis is:

A $-1 + \log_e 3$

B $-\frac{1}{3}e^{-3} - 3$

C $\frac{2092}{625}e$

D $\log_e 3 + \frac{1}{3e^9} + 8$

E $\log_e 3 + \frac{1}{3e^9} - 10$

11 The total area of the regions enclosed by the curve $y = 4\sin\left(\frac{x}{2} + \frac{\pi}{2}\right)$, the lines $x = -2\pi$ and $x = 2\pi$, and the x-axis is:

A -16

B 0

C 16

D $16 + 24\sqrt{2}$

E 32

12 The function f such that $f'(x) = -6 \sin 3x$ and $f(\frac{2\pi}{3}) = 3$ is:

A $-18 \cos 3x + 21$

B $-2 \cos 2x + 5$

C $-2 \sin 3x + 1$

D $2 \cos 3x + 1$

E $2 \sin 4x + 3$

13 If a curve with the equation $y = f(x)$ passes through the point (2, 0), and $f'(x) = (x - 2)^3$, then the equation of the curve is:

A $y = \dfrac{(x-2)^4}{4}$

B $y = \dfrac{(x-2)^4}{4} - 2$

C $y = \dfrac{(x-2)^3}{3} + \dfrac{5}{3}$

D $y = (x-2)^3 + 1$

E $y = 4 - 2(x-2)^4$

14 The area of the region between the graphs of $y = x^2 + 9$ and $y = 25 - x^2$ is given by:

A $2\int_{-\sqrt{8}}^{\sqrt{8}} 8 - x^2 \, dx$

B $2\int_{-\sqrt{8}}^{\sqrt{8}} x^2 - 8 \, dx$

C $\int_{-5}^{5} 25 - x^2 \, dx - \int_{-3}^{3} 9 + x^2 \, dx$

D $\int_{-\sqrt{8}}^{\sqrt{8}} 16 - x^2 \, dx$

E $\int_{-\sqrt{8}}^{\sqrt{8}} x^2 - 16 \, dx$

15 The value of $\int_0^{\pi} \cos(x^2 + 1)\, dx$ correct to three decimal places is:

A –3.142

B –0.345

C 0.000

D 1.571

E 3.142

16 A particle moves along a straight line such that its acceleration at time t is given by $6t - 4$ m/s^2. Initially the particle is at the point $O(x = 0)$ and has a velocity of –2 m/s^2. The position of the particle from O at time t is:

A $t^3 - 2t^2 - 2t$

B $t^3 - 2t^2$

C 6

D $t^3 - 2t^2 + 2$

E 0

17 A body moves in a straight line so that its acceleration in m/s^2 at time t seconds is given by $\frac{d^2x}{dt^2} = 6 - e^{-t}$. If the initial velocity of the body is 3 m/s, the velocity in m/s when $t = 2$ is:

A e^{-2}

B $2 + e^{-2}$

C $14 + e^{2}$

D $14 + e^{-2}$

E $14 - e^{-2}$

18 The average value of the function $y = \frac{1}{x}$ over the interval $[1, e]$ is:

A $\frac{1}{e - 1}$

B $e - 1$

C $\frac{1}{e}$

D 2

E $\frac{1}{1 - e}$

Short-answer questions (technology-free)

1 Find an antiderivative of each of the following:

a $\sin\left(5x + \frac{\pi}{6}\right)$

b $\frac{1}{3}\cos 6x$

c $e^{3x} + 5x$

d $5\cos 5x - 3e^{-2x+2}$

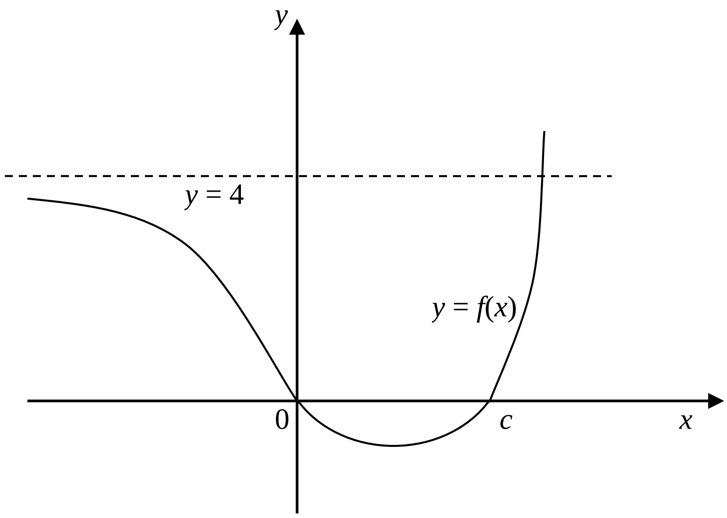
e $\sin(2x + 2) + 3$

f $\cos\left(3x + \frac{\pi}{4}\right)$

g $\frac{1}{2x + 3}$

2 Find $f(x)$ if $f'(x) = e^{\frac{x}{3}}$ and $f(0) = 1$

3 The graph of the function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = e^{2x} - 5e^{x} + 4$ is shown below:



The graph of f passes through the origin and the point $(c, 0)$.

a Find the exact value of c.

b Find the exact values of the coordinates of the turning point of the graph of f.

c Write an appropriate definite integral that determines the area of the region bounded by the graph of f and the x-axis, and find the exact value of this area.

4 **a** Differentiate $x \log_e(x)$ with respect to x.

b Hence or otherwise evaluate $\int_1^e \log_e(x)\ dx$.

5 **a** Differentiate $\log_e(x^2 + 1)$ with respect to x.

b Hence or otherwise evaluate $\int_0^1 \frac{3x}{x^2+1}\ dx$.

6 The curve with equation $y = f(x)$ has a gradient function with rule $f'(x) = 3x^2 + k$, where k is a constant, and has a turning point with coordinates $(-2, 6)$. Find:

a the value of k

b the rule $f(x)$

7 Given that $\int_0^1 f(x)\ dx = \int_1^5 f(x)\ dx = 5$, evaluate each of the following:

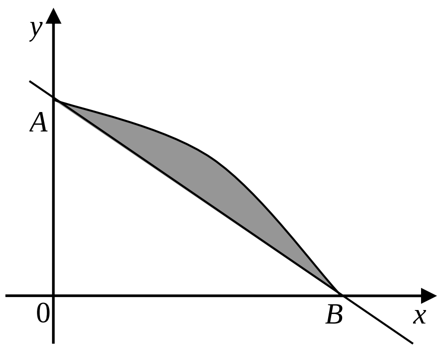
a $\int_0^1 f(x)\ dx + \int_5^1 f(x)\ dx$

b $\int_0^5 f(x)\ dx$

c $\int_1^5 [2f(x) + 5]\ dx$

8 Find the area of the region bounded by the graph of $y = \sin x$, the lines with equations $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ and the x-axis.

9 The graph of $y = \cos x$ is shown for $x \in \left[0, \frac{\pi}{2}\right]$.



a Find the equation of the straight line that passes through points A and B.

b Find the area of the shaded region.

10 A particle starts from rest 3 metres (to the right) from a fixed point and moves in a straight line with an acceleration of $a = 6t + 8$. Find its position and velocity at any time t seconds.

11 A particle moving in a straight line has acceleration of $(3 - 2t)$ m/s^2 at time t seconds ($t \geq 0$). If the particle starts at the origin O with a velocity of 4 m/s^2, find:

a the time when the particle comes to rest

b the position of the particle at the instant it comes to rest

c the acceleration at this instant

d the time when the acceleration is zero

e the velocity at this time.

12 Find the average value of each of the following functions for the stated interval.

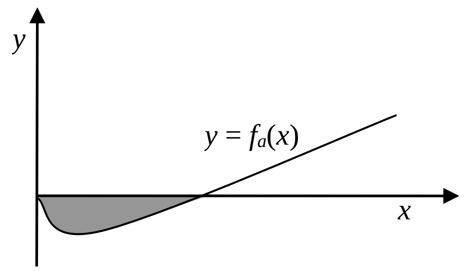
a $f(x) = \frac{1}{x}$, $\quad x \in [\,2, 2e]$

b $f(x) = \sin(x)$, $\quad x \in [\,0, \frac{3\pi}{2}]$

Extended-response questions

Consider the family of functions $f_a: [0, \infty) \to \mathbb{R}$, defined by

$f_a(x) = x - a\,x^{3/5}$ where a is a real number, $a > 0$.



1 $f_a(c) = 0$ and $c \neq 0$. Express c in terms of a.

2 Determine intervals on which f_a is a decreasing function and the intervals on which f_a is an increasing function.

3 Find the equations to the tangents to the graph of f_a at the point where the graph of f_a crosses the x-axis. What can be said about these tangents?

4 What is the range of f_a?

5 Find the exact value of the area of the shaded region.

Answers to Chapter 11 Test 1

Answers to multiple-choice questions

1 E

2 C

3 B

4 D

5 E

6 B

7 B

8 C

9 C

10 D

11 E

12 D

13 A

14 A

15 B

16 A

17 D

18 A

Answers to short-answer (technology-free) questions

1 **a** $\frac{-1}{5}\cos\left(5x+\frac{\pi}{6}\right)$

b $\frac{1}{18}\sin 6x$

c $\frac{1}{3}e^{3x}+\frac{5x^2}{2}$

d $\sin(5x)+\frac{3}{2}e^{2-2x}$

e $-\frac{1}{2}\cos(2x+2)+3x$

f $\frac{1}{3}\sin\left(3x+\frac{\pi}{4}\right)$

g $\frac{1}{2}\log_e(2x+3)$

2 $3e^{\frac{x}{3}} - 2$

3 a $\log_e(4)$

b $\left(\log_e\left(\frac{5}{2}\right), \frac{-9}{4}\right)$

c $-\int_0^{\log_e 4} e^{2x} - 5e^x + 4dx,\ -\left(8\log_e(2) - \frac{15}{2}\right)$

4 a $\log_e(x) + 1$

b 1

5 a $\frac{2x}{x^2+1}$

b $\frac{3\log_e 2}{2}$

6 a $k = -12$

b $x^3 - 12x - 10$

7 a 0

b 10

c 30

8 $\frac{\sqrt{3}}{2} - \frac{1}{2}$

9 a $y = \frac{-2}{\pi}x + 1$

b $1 - \frac{\pi}{4}$

10 Position = $t^3 + 4t^2 + 3$; velocity = $3t^2 + 8t$

11 a 4 seconds

b $\frac{56}{3}$ m

c -5 m/s^2

d $\frac{3}{2}$ s

e $\frac{25}{4}$ m/s

12 a $\frac{1}{2(e-1)}$

b $\frac{2}{3\pi}$

Answers to extended-response questions

1 $c = a^{5/2}$

2 decreasing on $(0, (\frac{3a}{5})^{\frac{5}{2}})$ and increasing on $((\frac{3a}{5})^{\frac{5}{2}}, \infty)$

3 $y = \frac{2}{5}(x - a^{\frac{5}{2}})$, all parallel

4 $[-\frac{2}{5}(\frac{3}{5})^{\frac{3}{2}}\ a^{\frac{5}{2}}, \infty)$

5 $\frac{a^5}{8}$