Year 12- Maths Methods: Topic 8 Integration Holiday Home work- Test

Name: ___________________________

Short answer — technology free

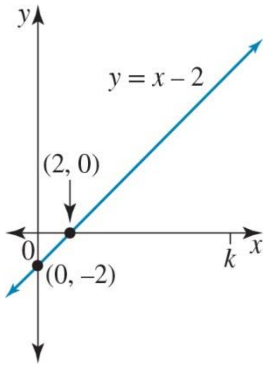
1 (a) Sketch the graph of the function 6

$f(x)=\frac{1}{x}$ for $x\in(0,6]$.

(b) Use the left end-point rule and rectangles 1 unit wide to estimate the area between the curve and the x-axis from $x=1$ to $x=5$.

(c) Use the right end-point rule and rectangles 1 unit wide to estimate the area between the curve and the x-axis from $x=1$ to $x=5$.

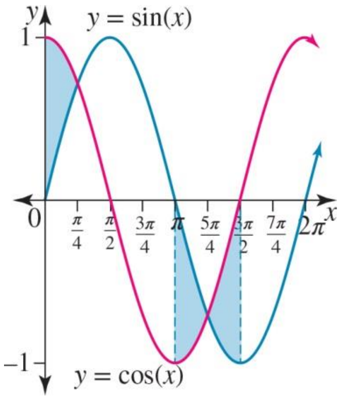
2 The graph of $y = x - 2$ is shown. 5



(a) Use your knowledge of the area of a triangle to find the value of k given that the area between the line and the x-axis from $x = 2$ to $x = k$ is 8 units^2.

(b) Use calculus to verify your answer to part **(a)**.

3 The graphs of $y = \sin(x)$ and $y = \cos(x)$ are shown. 4

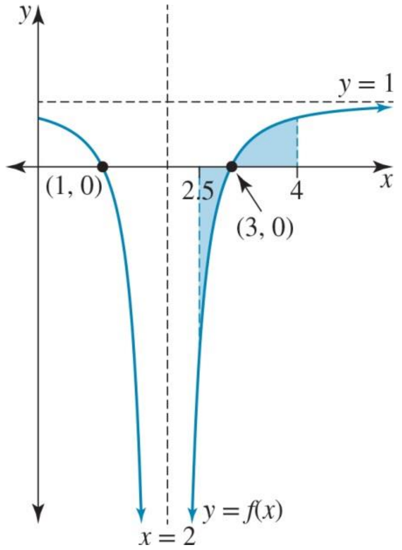


Find the exact area of the shaded region.

4 The graph of 4

$$f: R \setminus \{2\} \rightarrow R,\ f(x) = 1 - \frac{1}{(x-2)^2}$$

is shown.



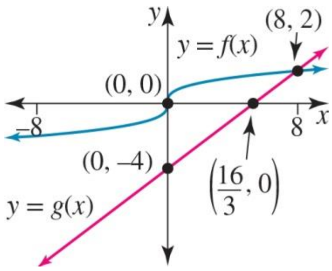
Find the area of the shaded region shown.

5 The graphs of the functions 4

$f : R \to R,\ f(x) = \sqrt[3]{x}$

and

$g : R \to R,\ g(x) = \dfrac{3}{4}x - 4$ are shown.



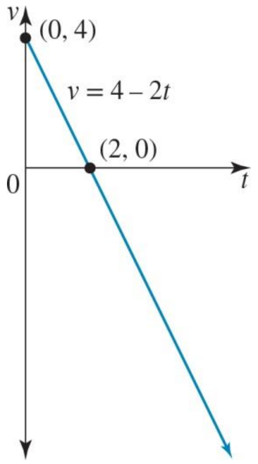
Find the area between the curve and the line

from $x = -8$ to $x = 8$.

6 The velocity of a particle is given by 7

$$v = \frac{dx}{dt} = 4 - 2t$$

where v is velocity in metres per second and t is time in seconds. The graph of v versus t is shown.



(a) Find the displacement of the particle if it was initially 12 metres to the right of the origin.

(b) What was the displacement of the particle after 6 seconds?

(c) Use calculus methods to find the distance travelled by the particle in the first 6 seconds.

Multiple choice

1 The approximate area between the curve $y=\cos(x)$, $x=0$ and $x=\frac{\pi}{2}$ using the left end-point rule with rectangles of width $\frac{\pi}{4}$ is given by:

A $\frac{\pi}{4}\left(1+\frac{1}{\sqrt{2}}\right)$

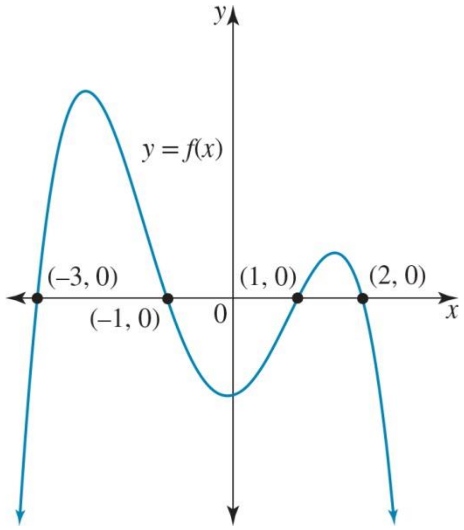
B $\frac{\pi}{4}\left(2+\frac{1}{\sqrt{2}}\right)$

C $\frac{\sqrt{2}\pi}{4}$

D $\frac{\pi}{2}$

E $\frac{\pi}{4}$

2 The area between the curve $y=f(x)$ and the x-axis from $x=-3$ to $x=2$ is given by:



A $\int_{-3}^{2} f(x)dx$

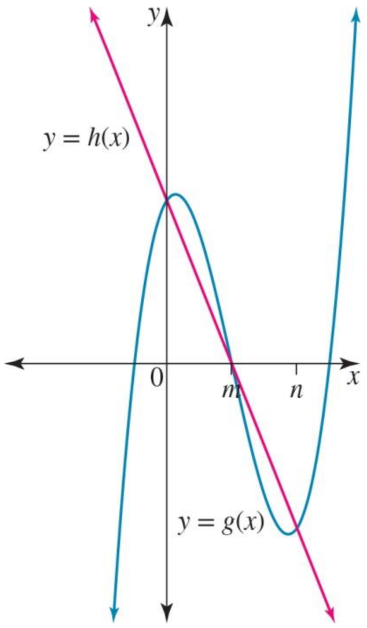
B $\int_{-3}^{-1} f(x)dx+\int_{-1}^{1} f(x)dx+\int_{1}^{2} f(x)dx$

C $\int_{-3}^{-1} f(x)dx+\int_{1}^{2} f(x)dx$

D $\int_{-3}^{-1} f(x)dx-\int_{-1}^{1} f(x)dx+\int_{1}^{2} f(x)dx$

E $\int_{-1}^{-3} f(x)dx-\int_{1}^{-1} f(x)dx+\int_{2}^{1} f(x)dx$

3 The area between the curves $y = g(x)$ and $y = h(x)$ from $x = 0$ to $x = n$ is given by:



A $\int_0^m \left(g(x) - h(x)\right) dx$
$+\int_m^n \left(h(x) - g(x)\right) dx$

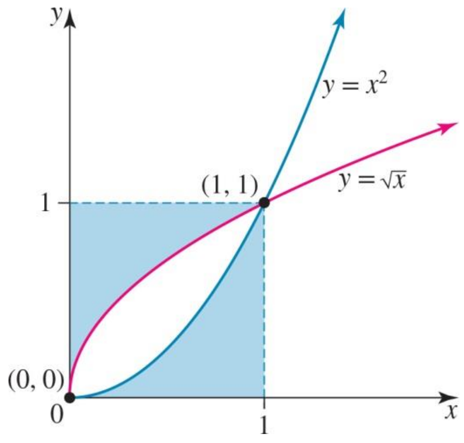
B $\int_0^m \left(g(x) - h(x)\right) dx$
$+\int_n^m \left(h(x) - g(x)\right) dx$

C $\int_n^m \left(g(x) - h(x)\right) dx$
$+\int_n^m \left(h(x) - g(x)\right) dx$

D $\int_0^m \left(h(x) - g(x)\right) dx$
$+\int_m^n \left(h(x) - g(x)\right) dx$

E $\int^n \left(g(x) - h(x)\right) dx$

4 The area of the shaded region is:



A $\frac{1}{3}$ units^2

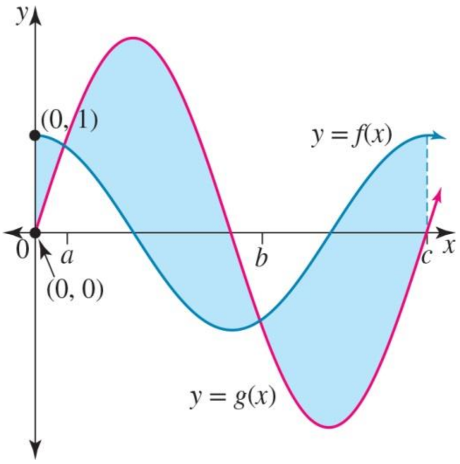
B $\frac{2}{3}$ units^2

C $\frac{1}{6}$ units^2

D $\frac{5}{6}$ units^2

E 1 units^2

5 The graphs of $f(x)=\cos(x)$ and $g(x)=2\sin(x)$ are shown.



The shaded region is given by:

A $\int_0^a \left(f(x)-g(x)\right)dx + \int_b^a \left(g(x)-f(x)\right)dx + \int_c^b \left(f(x)-g(x)\right)dx$

B $\int_0^c \left(g(x)-f(x)\right)dx$

C $\int_0^c \left(f(x)-g(x)\right)dx$

D $\int_0^a \left(f(x)-g(x)\right)dx + \int_a^b \left(g(x)-f(x)\right)dx + \int_b^c \left(g(x)-f(x)\right)dx$

E $\int_0^a \left(f(x)-g(x)\right)dx + \int_a^b \left(g(x)-f(x)\right)dx + \int_b^c \left(f(x)-g(x)\right)dx$

6 The exact area bounded by the curve $y=e^{-x}$, the x-axis, the y-axis and the line $x=-2$ is:

A e^2 units^2

B $1+e^2$ units^2

C $e^{-2}-1$ units^2

D e^2-1 units^2

E $1+e^{-2}$ units^2

7 An athlete initially has 2 litres of oxygen in her lungs. She then inhales oxygen at a rate defined by

$\frac{dq}{dt}=4-(t+0.5)^{-2}$ litres/minute,

where q is the quantity of oxygen inhaled in litres and t is the time in minutes. The amount of oxygen in her lungs is given by

A $q=4t+\frac{1}{t+0.5}+\frac{1}{2}$

B $q=4t-\frac{1}{t+0.5}+2$

C $q=4t-\frac{1}{t+0.5}+\frac{1}{2}$

D $q=4t+\frac{1}{t+0.5}$

E $q=4t+\frac{1}{t+0.5}+2$

8 A particle has a velocity, v cm/s, defined by $v = 4 - t^2$ where t is in seconds and $t \geq 0$. The displacement and acceleration of the particle, if it is initially at the origin, are:

A $x = \frac{1}{3}t^3 + 4t$, $a = -2t$

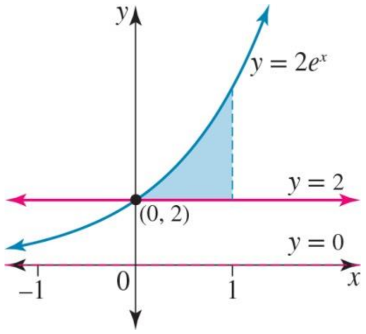
B $x = -\frac{1}{3}t^3 + 4t$, $a = 2t$

C $x = -\frac{1}{3}t^3 - 4t$, $a = -2t$

D $x = -\frac{1}{3}t^3 + 4t$, $a = -2t$

E $x = -\frac{1}{3}t^3 - 4t$, $a = 2t$

9 The graphs of $y = 2e^x$ and $y = 2$ are shown.



The area of the shaded region is given by:

A $\int_0^1 (2e^x - 2)dx$

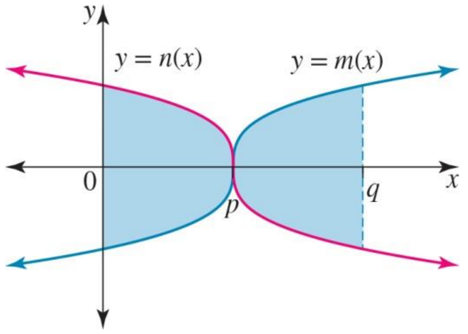
B $\int_0^1 (e^x - 2x)dx$

C $\int_1^0 (2e^x - 2)dx$

D $\int_0^1 1dx$

E $\int_0^1 (2 - 2e^x)dx$

10 The graphs of $y = m(x)$ and $y = n(x)$ are shown.



The area of the shaded region is given by:

A $\int_0^q \left(m(x) - n(x)\right) dx$

B $\int_0^q \left(n(x) - m(x)\right) dx$

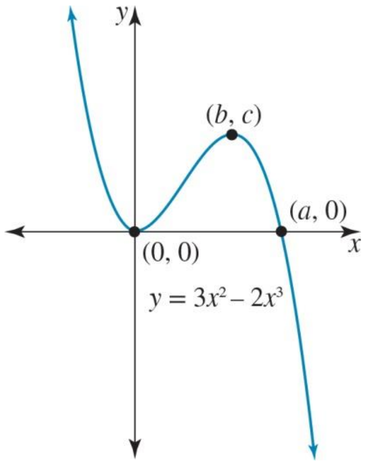
C $\int_0^p \left(m(x) - n(x)\right) dx + \int_p^q \left(m(x) - n(x)\right) dx$

D $\int_p^q m(x) dx + \int_0^p n(x) dx$

E $\int_0^p \left(n(x) - m(x)\right) dx + \int_p^q \left(m(x) - n(x)\right) dx$

Extended response

1 The graph of $y = 3x^2 - 2x^3$ is shown.



(a) Find the integer values of a, b and c.

(b) Find the area between the curve and the x-axis from $x = b$ to $x = 2$.

(a) The graph cuts the x-axis where $y = 0$:

$$3x^2 - 2x^3 = 0$$
$$x^2(3 - 2x) = 0$$
$$x = 0 \text{ or } x = \frac{3}{2}$$

So $(a,0) = \left(\frac{3}{2}, 0\right)$.

A turning point occurs where $\frac{dy}{dx} = 0$:

$$\frac{dy}{dx} = 6x - 6x^2$$
$$0 = 6x(1 - x)$$
$$x = 0 \text{ or } x = 1$$

When $x = 1$, $y = 3(1)^2 - 2(1)^3 = 1$.

So $(b, c) = (1,1)$.

(b) The required area is given by

$$\int_1^{\frac{3}{2}} (3x^2 - 2x^3)dx - \int_{\frac{3}{2}}^{2} (3x^2 - 2x^3)dx$$

$$= \left[x^3 - \frac{1}{2}x^4\right]_1^{\frac{3}{2}} - \left[x^3 - \frac{1}{2}x^4\right]_{\frac{3}{2}}^{2}$$

$$= \left(\left(\left(\frac{3}{2}\right)^3 - \frac{1}{2}\left(\frac{3}{2}\right)^4\right) - \left((1)^3 - \frac{1}{2}(1)^4\right)\right)$$
$$- \left(\left((2)^3 - \frac{1}{2}(2)^4\right) - \left(\left(\frac{3}{2}\right)^3 - \frac{1}{2}\left(\frac{3}{2}\right)^4\right)\right)$$

$$= \frac{27}{8} - \frac{81}{32} - \frac{1}{2} - \left(0 - \left(\frac{27}{8} - \frac{81}{32}\right)\right)$$

$$= \frac{27}{8} - \frac{81}{32} - \frac{1}{2} + \frac{27}{8} - \frac{81}{32}$$

$$= \frac{19}{16} \text{ units}^2$$

2 Heat escapes from a storage tank at a rate of kilojoules/day. The equation relating the heat loss over t days from July 1 is given by 8

$$\frac{dH}{dt} = 1 + \frac{4}{5}\sin\left(\frac{\pi t}{40}\right),\ 0 \le t \le 120$$

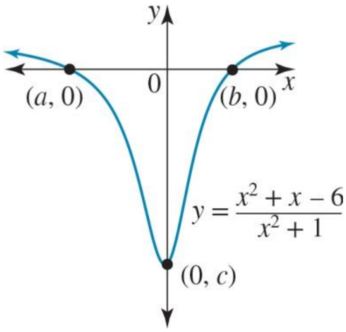
where H is the total accumulated heat lost.

(a) Find the maximum and minimum rates of heat lost over the 120 days.

(b) Sketch the graph of $\frac{dH}{dt}$ versus t for $0 \le t \le 120$.

(c) Calculate the amount of heat lost over the 120 days. Give your answer correct to the nearest kilojoule.

3 The graph of $y = \dfrac{x^2 + x - 6}{x^2 + 1}$ is shown. 8



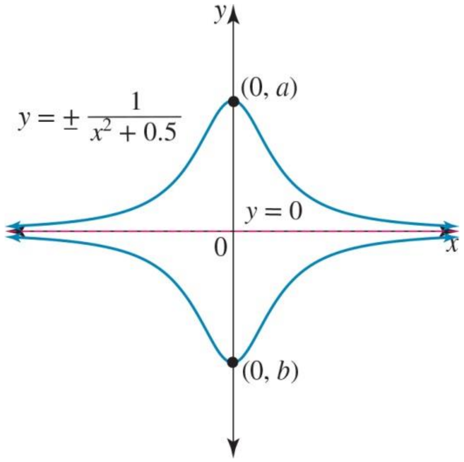
(a) The graph cuts the x-axis at $(a, 0)$ and $(b, 0)$. It cuts the y-axis at $(0, c)$. Find the integer values of a, b and c.

(b) The line $y = mx + d$ passes through the points $(-1, -3)$ and $(b, 0)$. Find the equation of the line and sketch this on the same set of axes used for $y = \dfrac{x^2 + x - 6}{x^2 + 1}$.

(c) Find the area between the curve and the line from $x = -1$ to $x = b$, giving your answer correct to 1 decimal place.

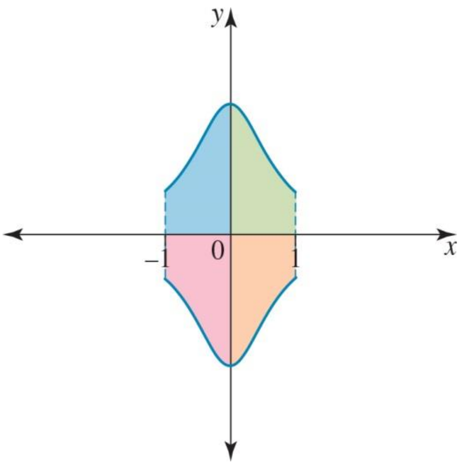
(Additional working space for question 3)

4 The graphs of $y = \pm\dfrac{1}{x^2 + 0.5}$ are shown. 7



(a) The graphs cut the y-axis at $(0, a)$ and $(0, b)$. Find the integer values of a and b.

(b) A graphic designer is using these curves to design a distinctive logo for a new client. The logo design is shown below. This colourful logo is to have the name of the company superimposed over the shaded region. All measurements are in centimetres.



(i) Find the area of the pale green region (the upper right region). Give your answer correct to 2 decimal places.

(continued)

(Question 4 continued)

(ii) Hence or otherwise, find the area of the whole shaded region, giving your answer correct to 2 decimal places.

(c) The logo is to be enlarged so that its length from a to b measures 1 metre. Determine the width of the logo.