

Use your text book to answer these questions.

Due Date: 16/07/2018

Multiple-choice questions

1 $\int_a^{a^2} 2x \, dx$ in terms of a is equal to:

A a^2

B $2a(a - 1)$

C $2a$

D a^3

E $a^2(a^2 - 1)$

2 $\int_0^2 3f(x) + 2 \, dx =$

A $3 \int_0^2 f(x) dx + 3x$

B $3 \int_0^2 f(x) dx + x$

C $3 \int_0^2 f(x) dx + 4$

D $3f'(x) + 4$

E $\int_0^2 f(x) dx + 4$

3 Given $\frac{dy}{dx} = ae^{-x} + 5$ and that when $x = 0$, $\frac{dy}{dx} = 7$ and $y = 1$, the value of y when $x = 2$ is

A $-\frac{2}{e^2} + 3$

B $-\frac{2}{e^2} + 13$

C $-\frac{3}{e^2} + 3$

D $2e^2 + 3$

Cambridge Senior Mathematical Methods AC/VCE Units 3 & 4
Online Teaching Suite Chapter 11 Integration: **Chapter test 1**

E $2e^2 + 10$

- 4 The curve with equation $y = a \sin\left(\frac{\pi}{4}x\right) + b$ passes through the points $(0, 3)$ and $(2, 5)$. The area of the region enclosed between this curve, the x -axis and the lines $x = 1$ and $x = 3$ is:

A $\frac{4\sqrt{2}}{\pi} + 3$

B $\frac{4\sqrt{2}}{\pi} + 9$

C $\frac{8\sqrt{2}}{\pi} + 3$

D $\frac{8\sqrt{2}}{\pi} + 6$

E $6 - \frac{8\sqrt{2}}{\pi}$

- 5 Given that $\frac{d}{dx}(x e^{2x}) = e^{2x} + 2x e^{2x}$, an antiderivative of $x e^{2x}$ is:

A $\frac{1}{2}e^{2x} + \frac{1}{2}x^2 e^{2x}$

B $\frac{1}{4}x^2 e^{2x}$

C $2e^{2x} + 2x e^{2x}$

D $-\frac{1}{2}e^{2x} + \frac{1}{2}x e^{2x}$

E $-\frac{1}{4}e^{2x} + \frac{1}{2}x e^{2x}$

- 6 Given that for $f(x) = x \sin x$, $f'(x) = \sin x + x \cos x$, an antiderivative of $x \cos x$ is:

A $x \sin x$

B $x \sin x + \cos x$

C $x \sin x - \cos x$

D $2 \cos x$

E $x \sin x - \sec^2 x$

- 7 If $f'(x) = e^{3x}$ and $f(0) = 1$ then $f(x) =$

A $\frac{1}{3}e^{3x} - \frac{2}{3}$

B $\frac{1}{3}e^{3x} + \frac{2}{3}$

- C** $\frac{1}{3} e^{3x} + 1$
- D** $3e^{3x} - 2$
- E** $3e^{3x} + 1$
- 8** The area of the region bounded by the curve $y = \sin \frac{x}{2}$, the x -axis and the line $x = \pi$ is:
- A** 0
- B** 1
- C** 2
- D** π
- E** 4
- 9** The total area of the regions bounded by the curve $y = \cos 2x$, the lines $x = 0$ and $x = \pi$, and the x -axis is:
- A** 0
- B** 1
- C** 2
- D** $\sqrt{3}$
- E** $\frac{1}{\sqrt{2}}$
- 10** The area of the region enclosed by the curve $y = -e^{3x} + 3$, the line $x = -3$ and the x -axis is:
- A** $-1 + \log_e 3$
- B** $-\frac{1}{3} e^{-3} - 3$
- C** $\frac{2092}{625} e$
- D** $\log_e 3 + \frac{1}{3e^9} + 8$
- E** $\log_e 3 + \frac{1}{3e^9} - 10$
- 11** The total area of the regions enclosed by the curve $y = 4 \sin \left(\frac{x}{2} + \frac{\pi}{2} \right)$, the lines $x = -2\pi$ and $x = 2\pi$, and the x -axis is:
- A** -16
- B** 0

- C 16
 D $16 + 24\sqrt{2}$
 E 32
- 12** The function f such that $f'(x) = -6 \sin 3x$ and $f(\frac{2\pi}{3}) = 3$ is:
- A $-18 \cos 3x + 21$
 B $-2 \cos 2x + 5$
 C $-2 \sin 3x + 1$
 D $2 \cos 3x + 1$
 E $2 \sin 4x + 3$
- 13** If a curve with the equation $y = f(x)$ passes through the point $(2, 0)$, and $f'(x) = (x - 2)^3$, then the equation of the curve is:
- A $y = \frac{(x-2)^4}{4}$
 B $y = \frac{(x-2)^4}{4} - 2$
 C $y = \frac{(x-2)^3}{3} + \frac{5}{3}$
 D $y = (x-2)^3 + 1$
 E $y = 4 - 2(x-2)^4$
- 14** The area of the region between the graphs of $y = x^2 + 9$ and $y = 25 - x^2$ is given by:
- A $2 \int_{-\sqrt{8}}^{\sqrt{8}} 8 - x^2 dx$
 B $2 \int_{-\sqrt{8}}^{\sqrt{8}} x^2 - 8 dx$
 C $\int_{-5}^5 25 - x^2 dx - \int_{-3}^3 9 + x^2 dx$
 D $\int_{-\sqrt{8}}^{\sqrt{8}} 16 - x^2 dx$
 E $\int_{-\sqrt{8}}^{\sqrt{8}} x^2 - 16 dx$

- 15 The value of $\int_0^{\pi} \cos(x^2 + 1) dx$ correct to three decimal places is:
- A -3.142
 - B -0.345
 - C 0.000
 - D 1.571
 - E 3.142
- 16 A particle moves along a straight line such that its acceleration at time t is given by $6t - 4$ m/s². Initially the particle is at the point $O(x = 0)$ and has a velocity of -2 m/s. The position of the particle from O at time t is:
- A $t^3 - 2t^2 - 2t$
 - B $t^3 - 2t^2$
 - C 6
 - D $t^3 - 2t^2 + 2$
 - E 0
- 17 A body moves in a straight line so that its acceleration in m/s² at time t seconds is given by $\frac{d^2x}{dt^2} = 6 - e^{-t}$. If the initial velocity of the body is 3 m/s, the velocity in m/s when $t = 2$ is:
- A e^{-2}
 - B $2 + e^{-2}$
 - C $14 + e^2$
 - D $14 + e^{-2}$
 - E $14 - e^{-2}$
- 18 The average value of the function $y = \frac{1}{x}$ over the interval $[1, e]$ is:
- A $\frac{1}{e-1}$
 - B $e-1$
 - C $\frac{1}{e}$
 - D 2
 - E $\frac{1}{1-e}$

Short-answer questions (technology-free)

1 Find an antiderivative of each of the following:

a $\sin\left(5x + \frac{\pi}{6}\right)$

b $\frac{1}{3} \cos 6x$

c $e^{3x} + 5x$

d $5 \cos 5x - 3e^{-2x+2}$

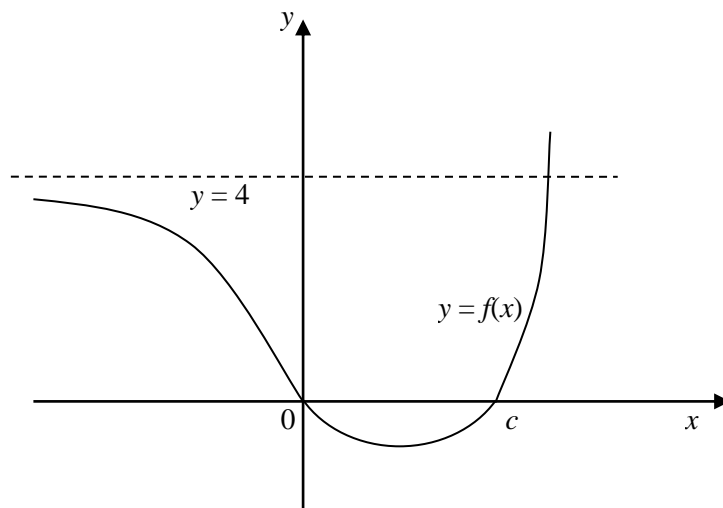
e $\sin(2x + 2) + 3$

f $\cos\left(3x + \frac{\pi}{4}\right)$

g $\frac{1}{2x+3}$

2 Find $f(x)$ if $f'(x) = e^{\frac{x}{3}}$ and $f(0) = 1$

3 The graph of the function $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^{2x} - 5e^x + 4$ is shown below:



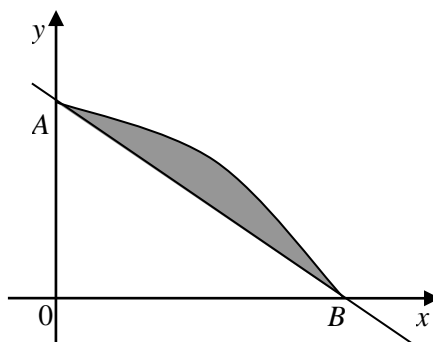
The graph of f passes through the origin and the point $(c, 0)$.

a Find the exact value of c .

b Find the exact values of the coordinates of the turning point of the graph of f .

c Write an appropriate definite integral that determines the area of the region bounded by the graph of f and the x -axis, and find the exact value of this area.

- 4 a Differentiate $x \log_e(x)$ with respect to x .
 b Hence or otherwise evaluate $\int_1^e \log_e(x) dx$.
- 5 a Differentiate $\log_e(x^2 + 1)$ with respect to x .
 b Hence or otherwise evaluate $\int_0^1 \frac{3x}{x^2 + 1} dx$.
- 6 The curve with equation $y = f(x)$ has a gradient function with rule $f'(x) = 3x^2 + k$, where k is a constant, and has a turning point with coordinates $(-2, 6)$. Find:
 a the value of k
 b the rule $f(x)$
- 7 Given that $\int_0^1 f(x) dx = \int_1^5 f(x) dx = 5$, evaluate each of the following:
 a $\int_0^1 f(x) dx + \int_5^1 f(x) dx$
 b $\int_0^5 f(x) dx$
 c $\int_1^5 [2f(x) + 5] dx$
- 8 Find the area of the region bounded by the graph of $y = \sin x$, the lines with equations $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ and the x -axis.
- 9 The graph of $y = \cos x$ is shown for $x \in \left[0, \frac{\pi}{2}\right]$.



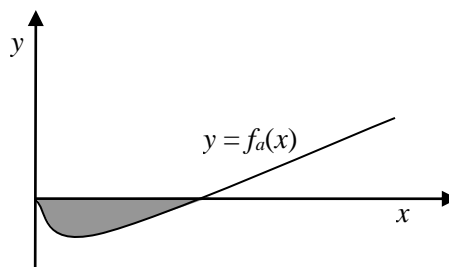
- a Find the equation of the straight line that passes through points A and B.
 b Find the area of the shaded region.

- 10** A particle starts from rest 3 metres (to the right) from a fixed point and moves in a straight line with an acceleration of $a = 6t + 8$. Find its position and velocity at any time t seconds.
- 11** A particle moving in a straight line has acceleration of $(3 - 2t)$ m/s² at time t seconds ($t \geq 0$). If the particle starts at the origin O with a velocity of 4 m/s², find:
- the time when the particle comes to rest
 - the position of the particle at the instant it comes to rest
 - the acceleration at this instant
 - the time when the acceleration is zero
 - the velocity at this time.
- 12** Find the average value of each of the following functions for the stated interval.
- $f(x) = \frac{1}{x}$, $x \in [2, 2e]$
 - $f(x) = \sin(x)$, $x \in [0, \frac{3\pi}{2}]$

Extended-response questions

Consider the family of functions $f_a: [0, \infty) \rightarrow \mathbb{R}$, defined by

$f_a(x) = x - ax^{3/5}$ where a is a real number, $a > 0$.



- $f_a(c) = 0$ and $c \neq 0$. Express c in terms of a .
- Determine intervals on which f_a is a decreasing function and the intervals on which f_a is an increasing function.
- Find the equations to the tangents to the graph of f_a at the point where the graph of f_a crosses the x -axis. What can be said about these tangents?
- What is the range of f_a ?
- Find the exact value of the area of the shaded region.

Answers to Chapter 11 Test 1

Answers to multiple-choice questions

- 1 E
- 2 C
- 3 B
- 4 D
- 5 E
- 6 B
- 7 B
- 8 C
- 9 C
- 10 D
- 11 E
- 12 D
- 13 A
- 14 A
- 15 B
- 16 A
- 17 D
- 18 A

Answers to short-answer (technology-free) questions

1 a $\frac{-1}{5} \cos\left(5x + \frac{\pi}{6}\right)$

b $\frac{1}{18} \sin 6x$

c $\frac{1}{3} e^{3x} + \frac{5x^2}{2}$

d $\sin(5x) + \frac{3}{2} e^{2-2x}$

e $-\frac{1}{2} \cos(2x + 2) + 3x$

f $\frac{1}{3} \sin\left(3x + \frac{\pi}{4}\right)$

g $\frac{1}{2} \log_e(2x + 3)$

2 $3e^{\frac{x}{3}} - 2$

3 a $\log_e(4)$

b $\left(\log_e\left(\frac{5}{2}\right), \frac{-9}{4} \right)$

c $-\int_0^{\log_e 4} e^{2x} - 5e^x + 4dx, -\left(8\log_e(2) - \frac{15}{2}\right)$

4 a $\log_e(x) + 1$

b 1

5 a $\frac{2x}{x^2 + 1}$

b $\frac{3 \log_e 2}{2}$

6 a $k = -12$

b $x^3 - 12x - 10$

7 a 0

b 10

c 30

8 $\frac{\sqrt{3}}{2} - \frac{1}{2}$

9 a $y = \frac{-2}{\pi}x + 1$

b $1 - \frac{\pi}{4}$

10 Position = $t^3 + 4t^2 + 3$; velocity = $3t^2 + 8t$

11 a 4 seconds

b $\frac{56}{3}$ m

c -5 m/s^2

d $\frac{3}{2}$ s

e $\frac{25}{4}$ m/s

12 a $\frac{1}{2(e-1)}$

b $\frac{2}{3\pi}$

Answers to extended-response questions

1 $c = a^{5/2}$

2 decreasing on $(0, (\frac{3a}{5})^{\frac{5}{2}})$ and increasing on $(\frac{3a}{5})^{\frac{5}{2}}, \infty)$

3 $y = \frac{2}{5}(x - a^{\frac{5}{2}})$, all parallel

4 $[-\frac{2}{5}(\frac{3}{5})^{\frac{3}{2}} a^{\frac{5}{2}}, \infty)$

5 $\frac{a^5}{8}$