Instructions to students: Read Chapter 11 in your text book. Go through examples and try to attempt all questions during your school holidays.

Due Date: 09-10-2018

Torque

Centre of mass (C of M)

Not on the course any more, but a very useful concept

Most structures are constructed of many different parts and are quite complex. They can be simplified by imagining that the gravitational force on the entire structure was acting at just a single point called the **Centre of Mass** or **Centre of Gravity** of the structure. The centre of mass is the balance point for a structure. Imagine a see-saw, the balance point is the middle of the beam. If the weight of the beam (structure) is not evenly distributed, we need to find the centre of mass using;

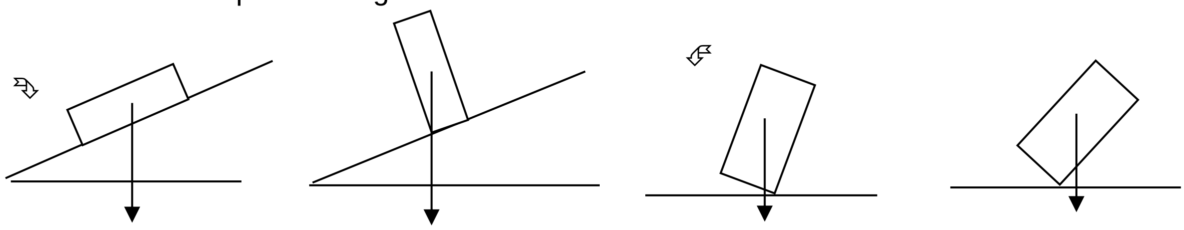
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots\ldots m_nx_n}{m_1 + m_2 + \ldots. m_n}$$

where m_1, m_2, …. are the masses of the system, and
x_1, x_2, ….. are the distances of the masses from a designated (by you) reference point.

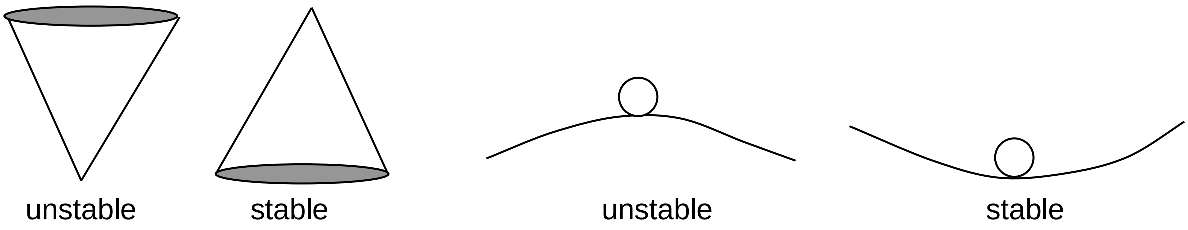
Stability of Structures

Not on the course any more, but an interesting concept

As the centre of mass is a balance point, an object is unstable if a vertical line through its centre of mass does not pass through its base.

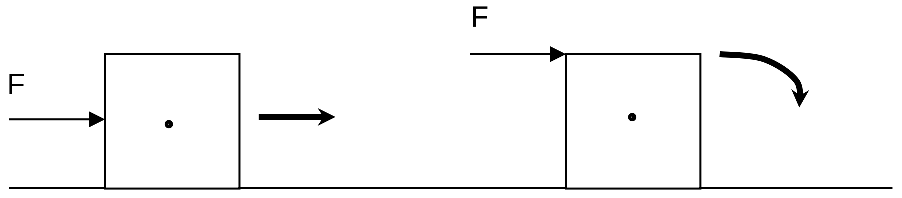


A structure is stable if the centre of mass is over the base. The width of the base and the height of the centre of mass affect the stability of a structure. To increase the stability, the centre of mass can be lowered and the width of the base increased.

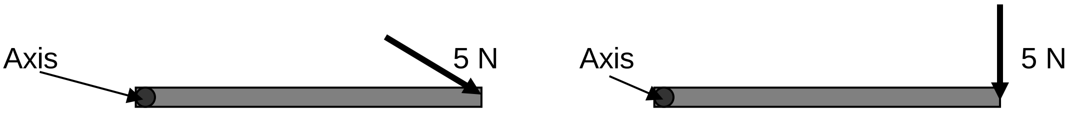


Torques (or bending moments)

When a force acts through the centre of mass of an object, it causes the translation of that of that object, in the direction of the net force. When the net force is applied through a point that is not the centre of mass, a torque is applied and rotation as well as translation occurs.



The turning force is most effective when it is acting at right angles to the rotating object. For example the turning force in the second diagram below produces more torque.

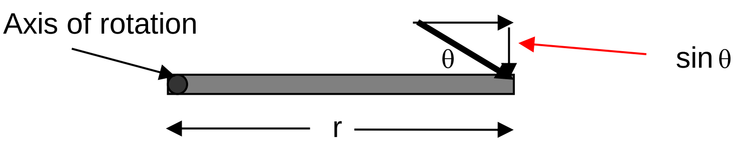


The product of the force applied, and the distance between the centre of mass, or the axis of rotation, and the point where the force is being applied gives the **magnitude of the torque**.

$$\tau = F \times r$$

The units of torque are Newton-metre (Nm). The sum of torques is found by adding all the products of forces and lever arms from that point. Clockwise torques are considered positive. A structure is stable, or rotationally balanced, if, the total clockwise moments = the total anticlockwise moments (or torques)

If the force acts at an angle θ to the object, then torque, $\tau = F \times r\sin\theta$. Which is actually the vertical component of the force. The horizontal component does not exert a turning effect on the beam.



Equilibrium

A structure is stable (in equilibrium) if the following two conditions are satisfied:

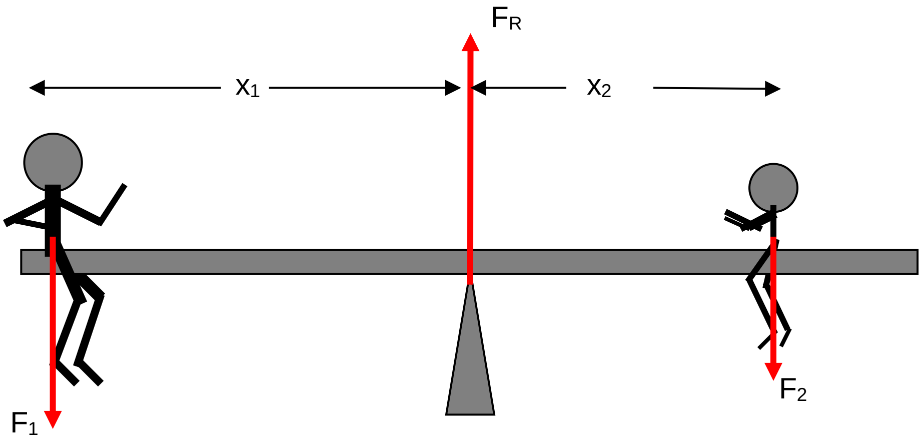
- the resultant or net force acting on the object must be zero, and
- the clockwise moments of the forces acting on the object must equal the anticlockwise moments.

In theory, when an object is in equilibrium it will stay in one position as long as it is not disturbed by any new influences. If a body tends to return immediately to its equilibrium position after a slight push, the object will be fairly stable. If it tends to move even more out of equilibrium, it is unstable. Stability depends generally on the relative position of the centre of mass of a structure (which will influence its 'susceptibility' to torques), while being in equilibrium depends more specifically on the absence of both net forces and torques (the moments of those forces).

Static Equilibrium

For a body to be in STATIC EQUILIBRIUM, it must have $\Sigma F = 0$ and $\Sigma \tau = 0$.

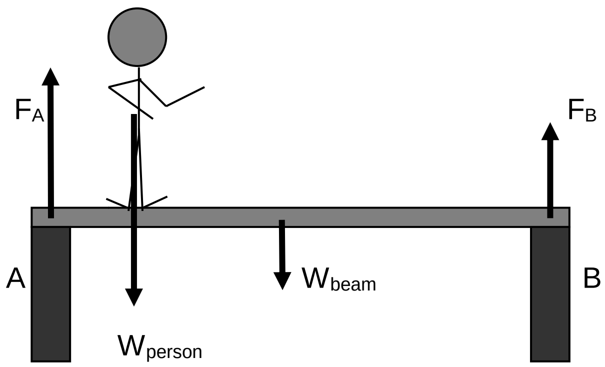
Seesaw.



For equilibrium: $\Sigma F = 0$ $\therefore$ $\Sigma F = F_1 + F_2 + F_R = 0$
For equilibrium: $\Sigma \tau = 0$ $\therefore$ $F_1 \times x_1 = F_2 \times x_2$

Beams

A person standing on the beam below, is part of a stable system that does not rotate or move in any direction



These forces are in equilibrium, so: $F_A + F_B = W_{person} + W_{beam}$.
The torques acting on the beam are also in equilibrium. If the torques are calculated using one end of the beam as the reference point, then either F_A or F_B is eliminated.
Taking torques around the left hand end:

$$\Sigma \tau = 0$$

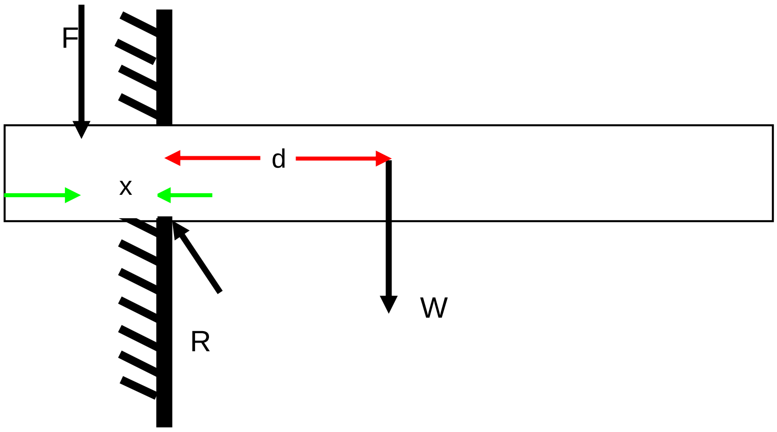
so $(F_A \times 0) + (W_{person} \times \text{distance}) + (W_{beam} \times \text{distance}) - (F_B \times \text{distance}) = 0$

If there was an unknown force acting somewhere, you usually find the torques about the point where this unknown force is acting, this effectively eliminates it from the calculations.

Cantilevers

A cantilever is part of a horizontal member that is unsupported at one end. Cantilevers must also fulfil the conditions of translational and rotational equilibrium if they are to remain stable.

If a cantilever is partially embedded in a wall, an upward reaction force will exist at the face of the wall as the cantilever pivots about this point. A downward force to resist rotation will exist at the middle of the embedded section. As always, the weight force of a uniform cross-sectioned cantilever will act at the middle.



For this cantilever to be in equilibrium, the sum of the torques must be equal to zero, and the sum of the forces must equal zero. If we resolve the moments around the point that it will pivot from, ie. where R is acting. The clockwise moments = W × d, and the anticlockwise moments are F × x.

So $W \times d = F \times x$.

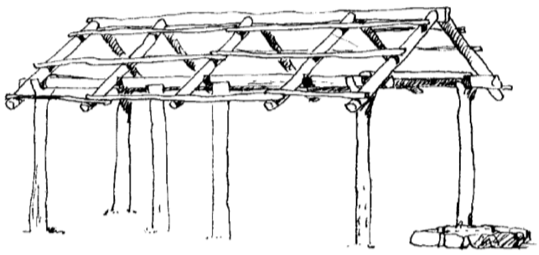
If we resolve the vertical forces, then the magnitude of R = F + W.
You can then use these two equations to work out the values of the forces.
The diving board and fence posts are also examples of cantilevers.

Strategy for solving problems involving torque

Questions regarding torque will often involve determining the value of two forces, so the solution will require generating two equations, which can then be solved simultaneously.
First, draw a diagram with all the forces acting on the structure. Label each force. If its size is given in the question, write the value, e.g. 10 N. If the size of the force is unknown, use a symbol such as ***F*** or ***R***.

1. Net force = zero. It is easier to break this up into two simpler tasks:
   a. Sum of forces up = Sum of forces down and
   b. Sum of forces left = Sum of forces right.
2. Net torque about any point = zero. Choose a point about which to calculate the torques. Any point will do, but it makes solving the problem easier if you choose a point through which an unknown force acts. The torque of this force about that point will be zero as its line of action passes through the point.

Sum of clockwise torques = Sum of anticlockwise torques.
Now you will have two equations with two unknowns; one equation from 2, above and one from 1.

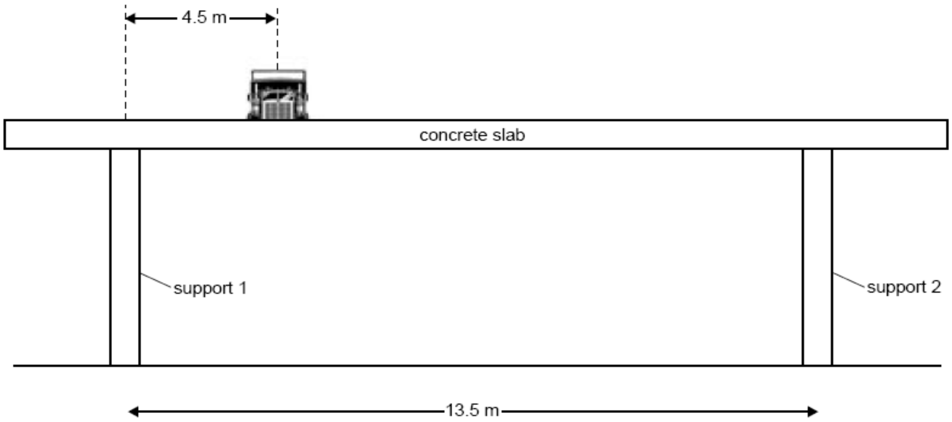


The frame of a bush hut is shown.

The roof is held up by four corner posts, each a forked tree trunk. In a detailed examination of such a structure, an engineer finds the force exerted by one of the roof beams on one side of the fork is 600 N in a direction perpendicular to the beam.
A detailed section is also shown.

Question 1 2001 Question 1

Calculate the magnitude of the torque on the left side of the fork about the point **X**, due to the force from the roof beam.

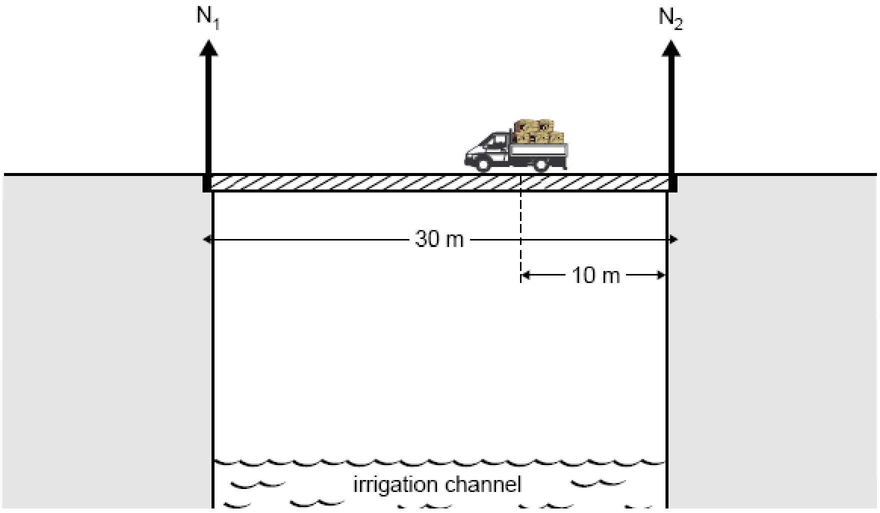


The figure above shows the front-on view of a loaded truck crossing a uniform concrete slab inside a building. The mass of the slab is 300 tonne, and the mass of the loaded truck is 50.0 tonne.
The centre of mass of the truck is 4.5 m from support 1.

Question 2 2003 Question 6

Calculate the magnitude of the contact forces supporting the slab when the centre of mass of the truck is 4.5 m from support 1.

The bridge over an irrigation channel is shown. The bridge can be considered as a uniform concrete beam of length 30 m and mass 20 tonnes. A heavily loaded small truck of mass 6 tonnes is pictured crossing the bridge.

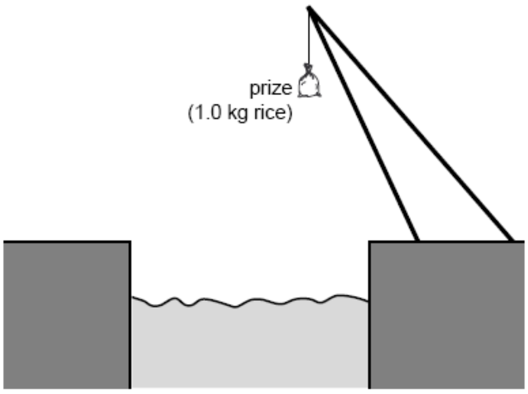


Question 3

2002 Question 3

Calculate the magnitude of each of the normal contact forces N_1 and N_2 at each end of the bridge when the centre of mass of the truck is 10 m from one end.

A survival course requires the participants to get a prize (1.0 kg of rice) suspended above a river. The figure shows the problem.



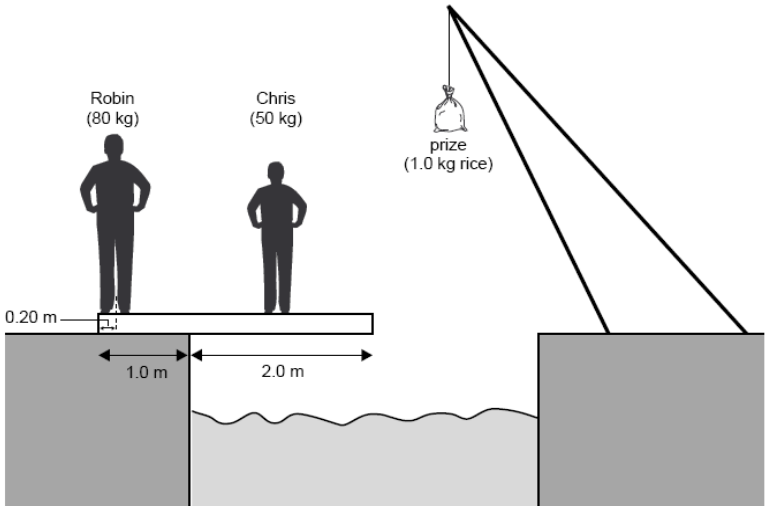
Question 4 2001 Question 7

Calculate the net force on the prize as it hangs.

Question 5 2001 Question 8

Calculate the tension in the string supporting the prize.

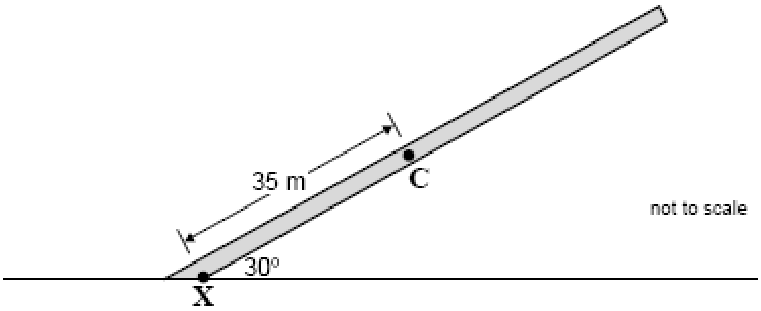
Two participants in the course, Chris and Robin, arrive on the scene. They are allowed to use a 40 kg, 3.0 m long plank to reach the prize. They set up the plank with 1.0 m on the bank, and Robin is standing so that his centre of mass is 0.20 m from the end. The figure below shows Chris walking out on the plank.



Question 6 2001 Question 9

Calculate how far from the bank Chris can safely walk.

The huge yellow beam which forms part of the Gateway to Melbourne can be modelled as a simple cantilever. It is a 70 m long steel box beam with a mass of 107 tonne, at an angle of 30º to the horizontal. This is shown below

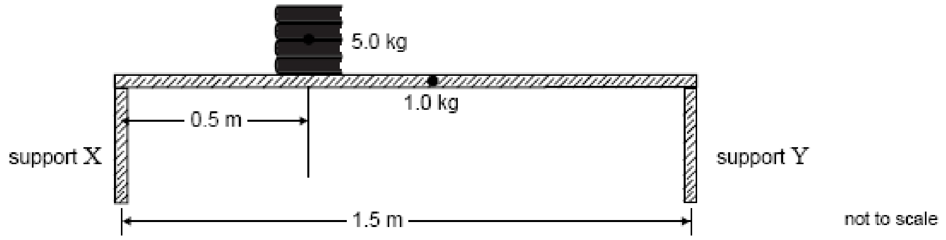


The centre of mass of the beam, C, can be considered to be 35 m from point X as shown.

Question 7 2000 Question 5

Calculate the torque that the weight of the beam produces about the anchor point X.

A uniform shelf of mass 1.0 kg rests on two supports X and Y. A pile of books, of mass 5.0 kg, is placed on the shelf as shown below.



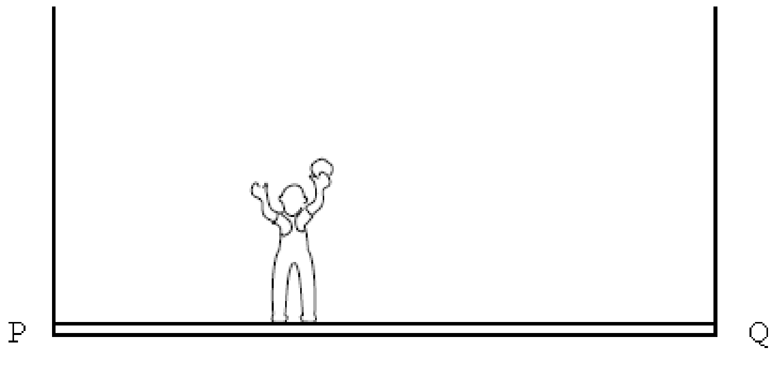
Question 8 1999 Question 3

Calculate the **total** upward force exerted on the shelf by the two supports.

Question 9 1999 Question 4

Calculate the magnitude of the upward force exerted by support X on the shelf.

A window cleaner of mass 60 kg walks along a plank of wood of mass 10 kg and of length 4.0 m from end P to end Q as shown below. The plank is suspended at each end by a light cable.



Question 10 1997 Question 11

As the cleaner moves from P to Q the tension in the cable connected to end Q can best be described as

A. varying from 0 N to 600 N.

B. varying from 50 N to 650 N.

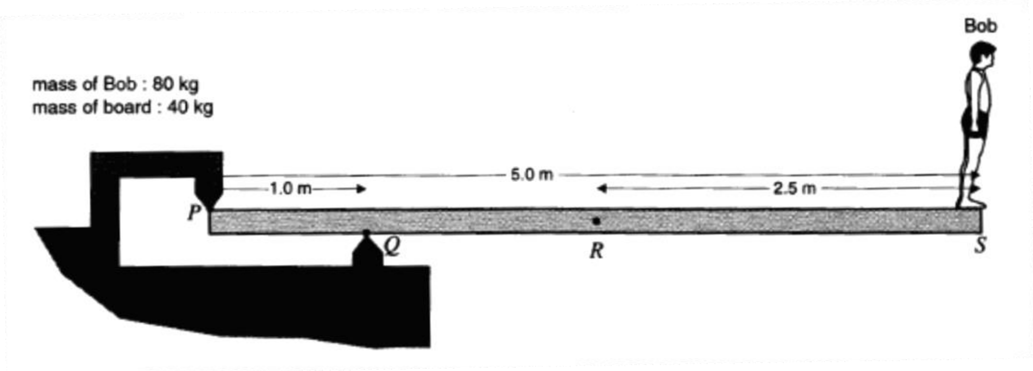
C. being constant as he moves.

D. varying from 100 N to 700 N.

Question 11 1997 Question 12

Find the tension in the cable connected to end P when the cleaner is 1.0 m from P.

Bob walked out and stood at the end (point, S) of a diving board as shown below.



Question 12 1996 Question 1

Indicate the direction of the contact forces acting on the board at points P and Q.

Question 13 1996 Question 2

Calculate the magnitude of the contact force acting on the board at the point P. Show your working.

Bob decides not to dive and walks back along the board.

Question 14 1996 Question 3

Which one of the following statements (**A** - **E**) best describes what would happen to the relative size of the contact forces acting on the board at points P and Q if Bob walked back down the board from S to R?

A. Both of the forces at P and Q would increase in magnitude.

B. Both of the forces at P and Q would decrease in magnitude.

C. The force at P would increase and that at Q would decrease in magnitude.

D. The force at P would decrease and that at Q would increase in magnitude.

E. There would be no change in the magnitude of the forces at P and Q