

Due Date: 08-10-2018

Multiple-choice questions

1 The gradient of the tangent to the curve $y = f(x)$ at the point $P(2, f(2))$ is given by

A $f(2) - h$

B $\frac{f(2+h) - f(2)}{2}$

C $\frac{f(2+h) - f(2)}{h}$

D $\lim_{h \rightarrow \infty} \frac{f(2+h) - f(2)}{h}$

E $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$

2 The gradient of the curve $y = x^3 + 3x^2 + 4x$ at the point where $x = 1$ is

A 0

B 4

C 13

D 8

E 16

3 If $y = x^4 - 5x^2 + 2x - 3$ then $\frac{dy}{dx}$ equals

A $x^4 - 5x^2 + 2$

B $4x^4 - 10x^2 + 2x - 3$

C $4x^4 - 10x^2 + 2x$

D $4x^3 - 10x + 2$

E $3x^4 - 4x^2 + x$

4 If $f(x) = x^2(x - 2)$ then $f'(1)$ equals

A -1

- B** 1
- C** 2
- D** -2
- E** 7

5 If $f(x) = (x - 5)^2$ then $f'(x)$ equals

- A** $x - 5$
- B** $x - 10$
- C** $2x$
- D** $2x + 25$
- E** $2x - 10$

6 If $y = \frac{2x^4 + 9x^2}{4x}$ then $\frac{dy}{dx}$ equals

- A** $\frac{2x^4}{4} + 18x$
- B** $\frac{8x^3 + 18x}{4}$
- C** $\frac{3x^2}{2} + \frac{9}{4}$
- D** $\frac{8x^3 + 18x}{4x}$
- E** $8x^3 + 18x$

7 For the function $y = -4x^3 - x^2 + 3x + 1$, the values of the y-coordinate and the gradient at $x = -1$ are

- A** -7 and -1
- B** 1 and -7
- C** 1 and -1
- D** 7 and -1
- E** -7 and 1

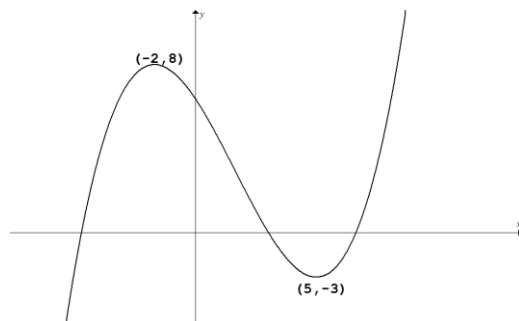
8 Given that $f(x) = 2x^3 + 3x^2 - 12x$, the value(s) of x for which $f'(x) = 0$ is/are

- A 0 and 3
- B 2
- C 2 and -1
- D -2 and 1
- E 0, 1 and 2

9 If $y = x^4 - 4x^3 - 8x^2$, the points at which the tangent to the curve is parallel to the x -axis are

- A 1, 0 and 4
- B 0 and 1
- C 1 and 4
- D 0 and 4
- E -1, 0 and 4

10 The graph of $y = f(x)$ is shown.



$f'(x) < 0$ for

- A $-2 \leq x \leq -5$
- B $-3 < x < 8$
- C $x < -2$ or $x > 5$
- D $-2 < x < 5$
- E $x < -3$ or $x > 8$

11 The derivative of $x^{-2} - \frac{1}{x}$ is

A $\frac{-2}{x} + \frac{1}{x^2}$

B $\frac{-2}{x^3} - \frac{1}{x^2}$

C $\frac{-2}{x} - \frac{1}{x^2}$

D $\frac{-2}{x^3} + 1$

E $\frac{-2}{x^3} + \frac{1}{x^2}$

12 $\int 3x^2 - 1 \, dx$ is equal to

A $6x$

B $x^2 - x + c$

C $\frac{1}{3}x^3 - x + c$

D $x^3 - x + c$

E $x^3 - x^2 + cx$

13 If $f(x) = 3x^2 - x$, then $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ is

A $6x$

B $6x - 1$

C $6x + 3h - 1$

D $3x - 1$

E $6x + h - 1$

14 If $\frac{dy}{dx} = -x + 4x^2 - 3$ and $y = 2$ when $x = 0$ then y is equal to

A $y = -1 + 8x$

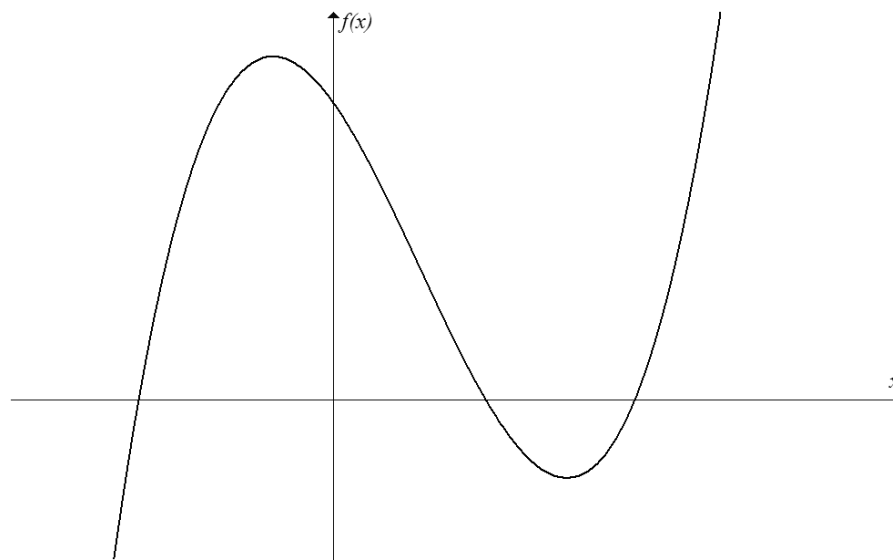
B $y = -\frac{1}{2}x^2 + 2x - 3x + 2$

C $y = -x^2 + \frac{4}{3}x^3 - 3x + 2$

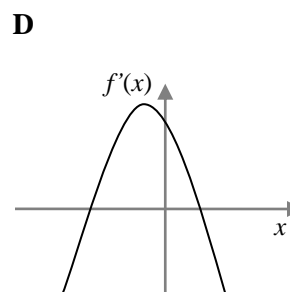
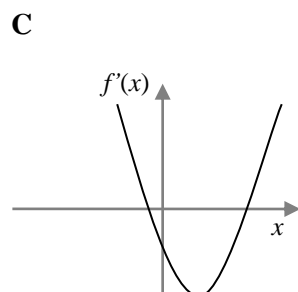
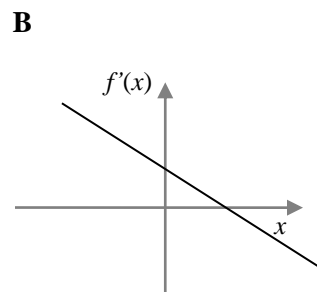
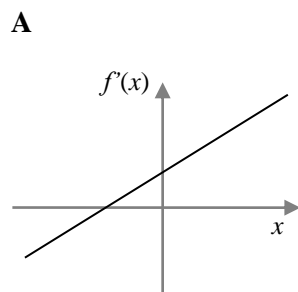
D $y = -\frac{1}{2}x^2 + \frac{4}{3}x^3 - 3x + 2$

E $y = -\frac{1}{2}x^2 + \frac{4}{3}x^3 - 3x - 2$

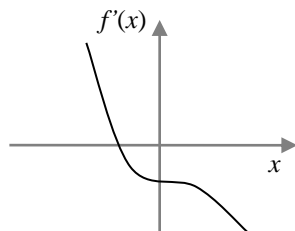
15 The graph of $y = f(x)$ is shown.



The graph of the gradient function, $f'(x)$, could be



E



Short-answer questions (technology-free)

1 Given that $f(x) = 2x^2 + 1$, find:

a $\frac{f(x+h) - f(x)}{h}$

b $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

2 Evaluate the following limits:

a $\lim_{x \rightarrow 3} \frac{5x+3}{x-1}$

b $\lim_{h \rightarrow 0} 3h - 5$

c $\lim_{x \rightarrow 2} 3x(x+4)$

d $\lim_{h \rightarrow 0} \frac{2h - h^2}{h}$

3 If $f(x) = (x-2)^2 + 4$, find:

a $f'(x)$

b $f'(2)$

c $\{x : f'(x) = 0\}$

4 Find $f'(x)$ for each of the following:

a $f(x) = 3x^3 - 2x + 4$

b $f(x) = 3(x+2)^2 - 2$

c $f(x) = \frac{8x^4 - 3x^3}{x^2}$

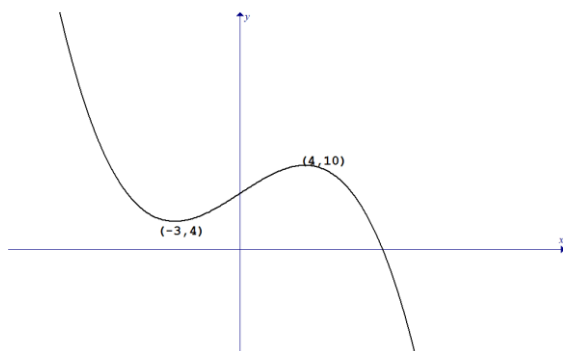
d $f(x) = 2x(x+1)^2$

e $f(x) = 2x^{-2} - x^{-4}$

f $f(x) = \frac{1}{x} - 3x$

- 5 a Find the y -coordinate and the gradient of the curve $y = x^4 - 3x^3 + 2x - 5$ at the point where $x = -2$.
- b Find the coordinates of the point(s) on the curve $y = x^3 - 3x$ at which the gradient is 24.

- 6 The graph of $f(x)$ is shown below.



- a For what values of x is the gradient negative?
- b For what values of x is the gradient positive?
- c Sketch the graph of the gradient function $f'(x)$.
- 7 Differentiate $f(x) = x^2 - 3x$ by first principles

- 8 Find each of the following:

a $\int 2x^4 - x + 1 \, dx$ b $\int (2x + 3)^2 \, dx$

Extended-response question

The area of a bushfire, A hectares, t hours after the fire has started is modelled by the rule

$$A = 90t^2 - 3t^3, t > 0.$$

- a Sketch a graph of the model over an appropriate domain, labelling all significant points.
- b Find $\frac{dA}{dt}$ for any time t .
- c Find the rate of change of the area of the bushfire at t equals:

i 4

ii 20

d Explain what happens after 20 hours.

e At what time is the rate of change of the area of the bushfire equal to 756 hectares per hour?

f After how many hours is the fire extinguished?

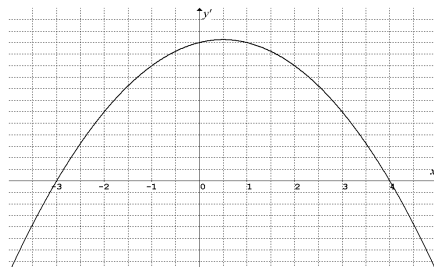
Answers

Multiple-choice questions

- | | | | | | | | | | |
|-----------|----------|-----------|----------|-----------|----------|-----------|----------|-----------|----------|
| 1 | E | 2 | C | 3 | D | 4 | A | 5 | E |
| 6 | C | 7 | B | 8 | D | 9 | E | 10 | D |
| 11 | E | 12 | D | 13 | B | 14 | D | 15 | C |

Short-answer questions (technology-free)

- | | | | | | | | | |
|----------|----------|----------------------------|----------|-----------------------------|----------|---------|----------|---|
| 1 | a | $4x + 2h$ | b | $4x$ | | | | |
| 2 | a | 9 | b | -5 | c | 36 | d | 2 |
| 3 | a | $2x - 4$ | b | 0 | c | $x = 2$ | | |
| 4 | a | $9x^2 - 2$ | b | $6x + 12$ | | | | |
| | c | $16x - 3$ | d | $f(x) = 6x^2 + 8x + 2$ | | | | |
| | e | $-4x^{-3} + 4x^{-5}$ | f | $f(x) = -\frac{1}{x^2} - 3$ | | | | |
| 5 | a | $y(-2) = 31; y'(-2) = -66$ | b | $(-3, -18), (3, 18)$ | | | | |
| 6 | a | $x < -3$ or $x > 4$ | b | $-3 < x < 4$ | | | | |
| | c | | | | | | | |



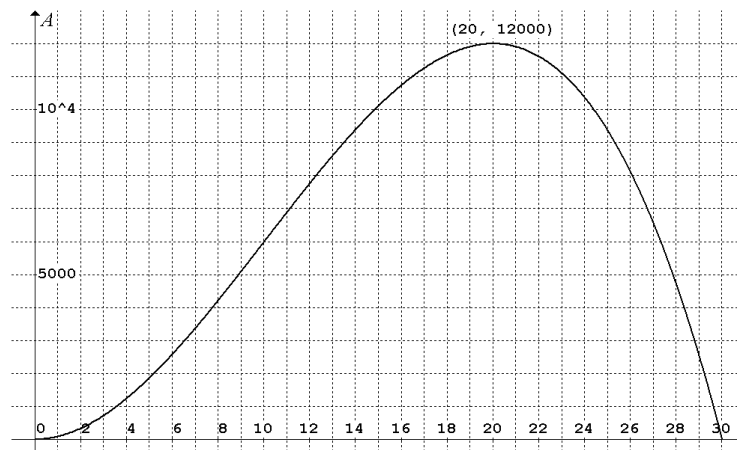
- 7** $2x - 3$

8 a $\frac{2}{5}x^5 - \frac{1}{2}x^2 + x + c$

b $\frac{4}{3}x^3 + 6x^2 + 9x + c$

Extended-response question

a



b $\frac{dA}{dt} = 180t - 9t^2$

c i 576 ha/h ii 0 ha/h

d After 20 hours the bushfire stops increasing in area and starts to diminish.

e At $t = 6$ and again at $t = 14$

f 30 hours