Topic 13: Differentiation and applications Test A

Name: _____

Short answer — technology-free

1 Evaluate: 3

$$\lim_{x \to 3} \frac{9 - x^2}{2x^2 - 4x - 6}$$

2 Calculate $f'(x)$ of the function 2

$f(x) = 3x^{\frac{2}{3}} + \sqrt{4x} + \sqrt{2}$.

3 Calculate the equation of the tangent to the curve 3

$y = 5x^3 - x + 8$ at the point where $x = -2$.

4 The curve $f(x) = x^6 + x^4 - 2$ has a stationary point at $(a, f(a))$.

3

a Determine the value of a.

b Identify the nature of the stationary point using the slope of the tangent in the neighbourhood of $(a, f(a))$.

2

5 $f : R \to R, f(x) = x^4 - 8x^2 - 2$

3

Determine the subset of the domain over which the function f is increasing.

6 A particle moves in a straight line such that its displacement, x metres, from a fixed origin at time t seconds is modelled by $x = t^2 - 6t + 1,\ t \geq 0$.

a Identify its initial position. 1

b After T seconds, the particle is momentarily at rest. Calculate T and the position of the particle at this time. 3

c What is the average speed of the particle over the first T seconds? 1

Multiple choice

1 At the point where $x = 8$, the curve $y = ax^{\frac{4}{3}}$ has a gradient of 48. The value of a is:

A 72
B 18
C 9
D 4.5
E 3

2 If $y = \dfrac{9+x^4}{4x^2}$, $x \neq 0$, then $\dfrac{dy}{dx}$ equals:

A $-\dfrac{9}{4x}+\dfrac{x}{2}$

B $\dfrac{9}{8x}+\dfrac{x}{2}$

C $\dfrac{9}{4x^3}+4x^3$

D $-\dfrac{9}{2x^3}+\dfrac{x}{2}$

E $\dfrac{72}{x^3}-4x^2$

3 The x-coordinates of the stationary points of the curve $f(x) = \dfrac{1}{3}x^3 + 4x^2 + 7x - 66$ are:

A $x = 3$

B $x = 1,\ x = 6$

C $x = 1,\ x = 7$

D $x = 7,\ x = 8$

E $x = -7,\ x = -1$

4 The following is known about the curve $y = f(x)$: $f'(x) < 0$ if $x < -5$; $f'(-5) = 0$, $f(-5) = 8$ and $f'(x) < 0$ if $x > -5$.
Which of the following is a correct statement about the curve $y = f(x)$?

A The curve has a minimum turning point at $(-5, 8)$.

B The curve has a maximum turning point at $(-5, 8)$.

C The curve has a stationary point of inflection at $(-5, 8)$.

D The curve has a global minimum at the point $(8, -5)$.

E The curve has a global maximum at the point $(8, -5)$.

5 When first planted, an oregano plant covers an area of 9 cm^2. This area grows according to the rule $A = \sqrt{3t} + 9$, where A cm^2 is the area covered by the oregano plant after t months. The rate at which the area is growing after 3 months is:

A $\dfrac{1}{6}$ cm^2 per month

B $\dfrac{1}{2}$ cm^2 per month

C $\sqrt{3}$ cm^2 per month

D $9\frac{1}{6}$ cm^2 per month

E 12 cm^2 per month

6 A circle has area A and radius r. The rate of change of the area with respect to its radius when its circumference is 10 cm is, in cm^2/cm:

A $\dfrac{dA}{dr} = 10$

B $\dfrac{dA}{dr} = 10\pi$

C $\dfrac{dA}{dr} = 20\pi$

D $\dfrac{dA}{dt} = 100\pi$

E $\dfrac{dr}{dA} = 20\pi$

7 $f : [0, 10] \to R,\ f(x) = (x-4)^2 + 20$
The absolute maximum and minimum values of the function f are:

A Maximum 10, minimum 4

B Maximum 10, minimum 0

C Maximum 20, minimum 4

D Maximum 56, minimum 20

E Maximum 36, minimum 20

8 $h(x) = \begin{cases} (2+x)^2, & x < -2 \\ 2, & x = -2 \\ x^2 - 4, & x > -2 \end{cases}$

$\lim\limits_{x \to -2} h(x)$ is:

A -4
B 0
C 2
D 16
E The limit does not exist.

9 The tangent drawn to which point on the curve $y = x^2 + 5x$ is parallel to the line $3x - y = 6$?

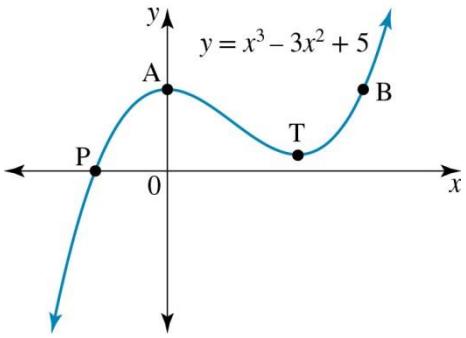
A $(-4, -18)$
B $(-4, -4)$
C $(-1, 3)$
D $(-1, -9)$
E $(-1, -4)$

10 The gradient of the tangent to the curve $y = 3\sqrt[3]{x}$ at the point where $x = 8$ is:

A 6
B $-\frac{1}{6}$
C $\frac{1}{4}$
D -4
E $\frac{1}{8}$

Extended response

1	The curve $y = 2x^3 + ax^2 + bx$ has a stationary point at $(2, -4)$. **a** Set up a pair of simultaneous equations for a and b. **b** Show that $a = -7$ and $b = 4$. **c** Determine the coordinates of the other stationary point. **d** Sketch the curve $y = 2x^3 - 7x^2 + 4x$, showing its turning points and identifying the exact coordinates of any intercepts with the coordinate axes.		2 1 2 3

2	The curve $f(x) = x^3 - 3x^2 + 5$ has a maximum turning point at A, a minimum turning point at T and an x-intercept at P. The points A and B have the same y-coordinate.  **a** Give the coordinates of the points A, T and B. **b** Calculate the angle $\theta°$ with which the tangent to the curve at B cuts the x-axis. **c** Determine the coordinates of the second point on the curve where the tangent will also cut the x-axis at an angle of $\theta°$. **d** An estimate of the x-coordinate of the point P is $x_0 = -1$. Use one iteration of the Newton–Raphson method to obtain a more accurate estimate for the x-coordinate of point P, expressing your answer correct to 2 decimal places.	·	4 2 2 2

3	A container in the shape of a square-based rectangular prism has a volume of $512\ \text{cm}^3$. Let the length of each side of the square base be x cm and the height of the container be h cm. **a** Express h in terms of x. **b** Show that the total surface area, $A\ \text{cm}^2$, of the container is $A = 2x^2 + \frac{2048}{x}$. **c** Use calculus to obtain the values of the base side length and height for which the total surface area is the smallest. You are not required to explain why A is the smallest with those values.		1 2 4

4	Consider the function $f(x)=\begin{cases}4x-3, & x\le 1\\ \dfrac{a}{x^2}, & x>1\end{cases}$ **a** Determine the value of a for $f(x)$ to be continuous at $x=1$. **b** Using the value for a obtained in part **a**, explain whether $f(x)$ is differentiable at $x=1$. **c** Using the value for a obtained in part **a**, form the rule for $f'(x)$.		2 3 1