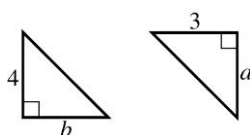


Instructions: Go through these answers to the three work sheets and use them to answer the questions to Test A on Deductive Geometry as your holiday homework. Hand this test to Mr Fernando when you come back to begin term 4.

WorkSHEET: Deductive geometry I Answers

Name: _____

- 1** Find the values of the pronumerals in the following pair of congruent triangles.



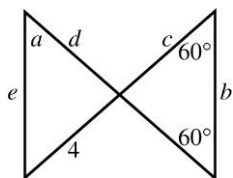
Congruent triangles mean all corresponding sides and all corresponding angles equal.

Therefore:

$$a = 4$$

$$b = 3$$

- 2** Find the values of the pronumerals in the following pair of congruent triangles.



Congruent triangles mean all corresponding sides and all corresponding angles equal.

Therefore both triangles are equilateral triangles. Therefore:

$$a = 60^\circ$$

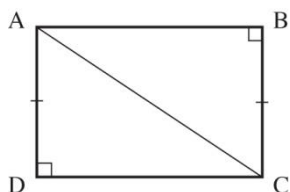
$$b = 4$$

$$c = 4$$

$$d = 4$$

$$e = 4$$

- 3** Prove that $\triangle ABC \cong \triangle CDA$.



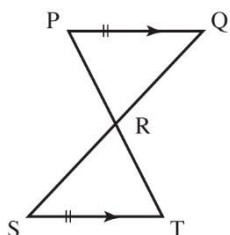
$AD = CB$ (given)

$\angle ADC = \angle CBA = 90^\circ$ (given)

AC is common.

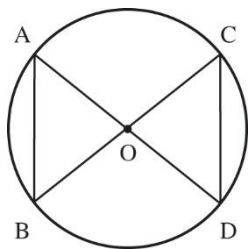
$\therefore \triangle ABC \cong \triangle CDA$ (RHS)

- 4 Prove that $\triangle PQR \cong \triangle TSR$ if $PQ = TS$ and $PQ \parallel ST$.



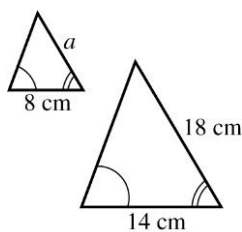
$PQ = TS$ (given)
 $\angle PQR = \angle TSR$ (Alternate angles equal as $PQ \parallel ST$.)
 $\angle QPR = \angle STR$ (Alternate angles equal as $PQ \parallel ST$.)
 $\therefore \triangle PQR \cong \triangle TSR$ (ASA)

- 5 AD and BC are diameters of a circle centred at O. Prove that $\triangle AOB \cong \triangle DOC$.



$AO = DO$ (Radii of same circle are equal.)
 $BO = CO$ (Radii of same circle are equal.)
 $\angle AOB = \angle DOC$ (Vertically opposite angles are equal.)
 $\therefore \triangle AOB \cong \triangle DOC$ (SAS)

- 6 Find the value of the pronumeral, a , in the pair of similar triangles.

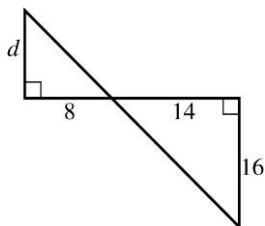


$$\frac{a}{18} = \frac{8}{14}$$

$$a = \frac{8}{14} \times 18$$

$$a = 10\frac{2}{7} \text{ cm}$$

- 7 Find the value of the pronumeral, d , in the figure below.

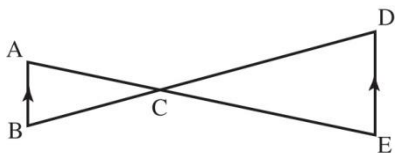


$$\frac{d}{16} = \frac{8}{14}$$

$$d = \frac{8}{14} \times 16$$

$$d = 9\frac{1}{7}$$

8 $AB \parallel DE$. Prove that $\triangle ABC \sim \triangle EDC$.



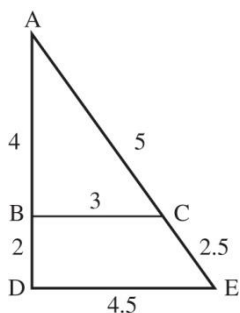
$\angle ACB = \angle ECD$ (Vertically opposite angles are equal.)

$\angle BAC = \angle DEC$ (Alternate angles equal as $AB \parallel DE$.)

$\angle ABC = \angle EDC$ (Alternate angles equal as $AB \parallel DE$.)

$\therefore \triangle ABC \sim \triangle EDC$ (Equiangular or AAA)

9 Prove that $\triangle ABC \sim \triangle ADE$.



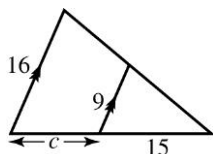
$$\frac{AD}{AB} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{DE}{BC} = \frac{4.5}{3} = \frac{3}{2}$$

$$\frac{AE}{AC} = \frac{7.5}{5} = \frac{3}{2}$$

$\therefore \triangle ABC \sim \triangle ADE$ (SSS)

10 Find the value of the pronumeral, c , in the figure below.



$$\frac{c+15}{15} = \frac{16}{9}$$

$$9(c+15) = 15 \times 16$$

$$9c + 135 = 240$$

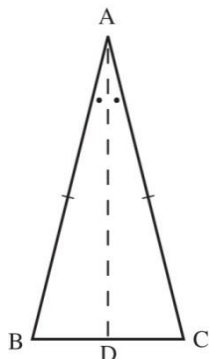
$$9c = 105$$

$$c = 11\frac{2}{3}$$

WorkSHEET: Deductive geometry II Answers

Name: _____

11 Prove that $\angle ABC = \angle ACB$.

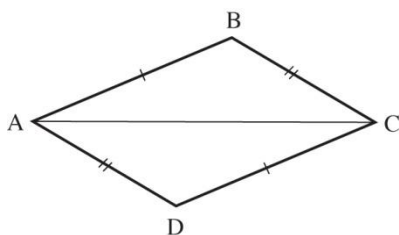


$AB = AC$ (given)
 $\angle BAD = \angle CAD$ (given)
 AD is common.
 $\therefore \triangle BAD \cong \triangle CAD$ (SAS)
 $\therefore \angle ABC = \angle ACB$ (Corresponding angles in congruent triangles are equal.)

12 Prove that each angle in an equilateral triangle is 60° .

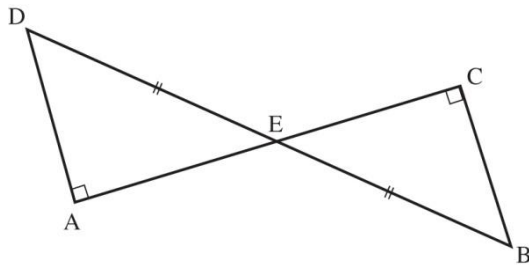
Let one angle be x .
 Since triangle is equilateral, all angles are equal.
 $x + x + x = 180^\circ$
 (Angle sum of any triangle is 180°)
 $3x = 180^\circ$
 $x = 60^\circ$
 \therefore Each angle is 60° .

13 In the figure below, prove that $\angle BAC = \angle DCA$



$AB = CD$ (given)
 $BC = DA$ (given)
 AC is common.
 $\therefore \triangle ABC \cong \triangle CDA$ (SSS)
 $\angle BAC = \angle DCA$ (Corresponding angles in congruent triangles are equal.)

- 14** In the figure below, prove that DB bisects AC.



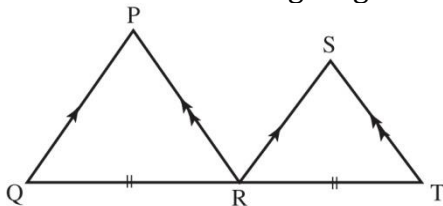
DE = BE (given)
 $\angle DAE = \angle BCE$ (given)
 $\angle DEA = \angle BEC$ (Vertically opposite angles are equal.)
 $\therefore \triangle ADE \cong \triangle CBE$ (ASA)
 $\therefore AE = CE$ (Corresponding sides in congruent triangles are equal.)
 \therefore DB bisects AC.

- 15** The following question appeared in a Maths test.
 In $\triangle PQR$ and $\triangle SRT$,

$QP \parallel RS$, $RP \parallel TS$ and $QR = RT$

Prove that $\triangle PQR \cong \triangle SRT$.

Andrew drew the following diagram.



Answer the question, and comment on Andrew's diagram.

QR = RT (given)
 $\angle PQR = \angle SRT$ (Corresponding angles equal as $QP \parallel RS$.)
 $\angle QPR = \angle PRS$ (Alternate angles equal as $QP \parallel RS$.)
 and $\angle PRS = \angle RST$ (Alternate angles equal $RP \parallel TS$.)
 $\therefore \angle QPR = \angle RST$
 $\therefore \triangle PQR \cong \triangle SRT$ (ASA)

In Andrew's diagram, $\triangle PQR$ and $\triangle SRT$ should be the same size. Also, it cannot be assumed that QRT is a straight line.

- 16** What is the definition of a parallelogram?

A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

- 17** True or false?

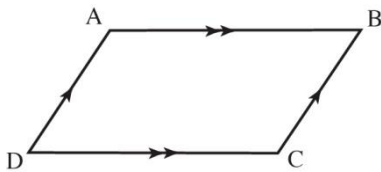
- (a) A trapezium has 2 equal sides. (a) False
- (b) The diagonals in a rhombus are equal in length. (b) False
- (c) The diagonals in a square bisect the angles they pass through. (c) True

-
- 18** Name four quadrilaterals with opposite angles equal. Parallelogram, rhombus, rectangle and square.
-
- 19** Name any quadrilaterals with diagonals that bisect each other. Parallelogram, rhombus, rectangle and square.
-
- 20** Name three quadrilaterals with diagonals that are perpendicular to each other. Square, rhombus, kite.
-

WorkSHEET: Deductive geometry III Answers

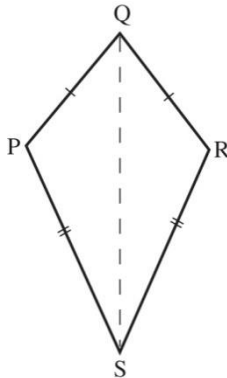
Name: _____

- 21** Prove that if one angle in a parallelogram is 90° , the quadrilateral is a rectangle.



$\angle ADC = 90^\circ$ (given)
 $\therefore \angle DAB = 90^\circ$ (Co-interior angles are supplementary as $AB \parallel DC$.)
 $\therefore \angle ABC = 90^\circ$ (Co-interior angles are supplementary as $AD \parallel BC$.)
 $\therefore \angle BCD = 90^\circ$ (Co-interior angles are supplementary as $AB \parallel DC$.)
 \therefore All angles are 90° .
 \therefore ABCD is a rectangle.

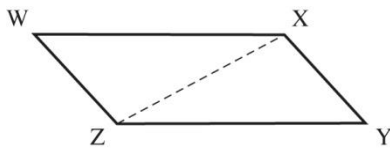
- 22** Use congruence to prove that the main diagonal of a kite bisects the angles it passes through.



$QP = QR$ (given)
 $PS = RS$ (given)
 QS is common.
 $\therefore \triangle QPS \cong \triangle QRS$ (SSS)
 $\therefore \angle PQS = \angle RQS$
 and $\angle PSQ = \angle RSQ$ (Corresponding angles in congruent triangles are equal.)

\therefore The main diagonal of a kite bisects the angles it passes through.

- 23** Prove that the opposite sides of a parallelogram are equal.

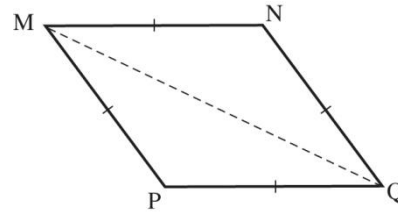


$WX \parallel ZY$ and $WZ \parallel XY$ since $WXYZ$ is a parallelogram.
 $\therefore \angle WZX = \angle YXZ$ (Alternate angles equal as $WZ \parallel XY$.)
 and $\angle WXZ = \angle YZX$ (Alternate angles equal as $WX \parallel ZY$.)

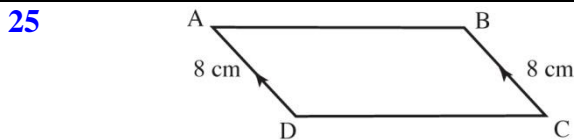
ZX is common.
 $\therefore \triangle WZX \cong \triangle YXZ$ (ASA)
 $\therefore WX = YZ$ and $WZ = YX$ (Corresponding sides in congruent triangles are equal.)

\therefore Opposite sides of a parallelogram are equal.

- 24** Prove that the main diagonal of a rhombus bisects the angles it passes through.



$MP = MN$ (Sides in a rhombus are equal.)
 $PQ = NQ$ (Sides in a rhombus are equal.)
 MQ is common.
 $\therefore \triangle MPQ \cong \triangle MNQ$ (SSS)
 $\therefore \angle PMQ = \angle NMQ$
 and $\angle PQM = \angle NQM$ (Corresponding angles in congruent triangles are equal.)
 $\therefore MQ$ bisects the angles it passes through.

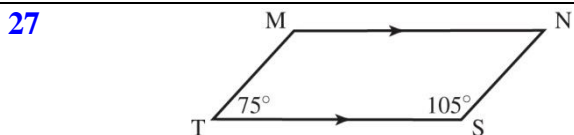


What sort of quadrilateral is ABCD?
Give reasons.

In ABCD,
 $AD \parallel BC$ (given)
 $AD = BC$ (given)
 \therefore One pair of opposite sides is equal and parallel.
 \therefore ABCD is a parallelogram.

- 26** True or false?
 (a) A quadrilateral with four equal sides must be a square.
 (b) A parallelogram has equal diagonals.

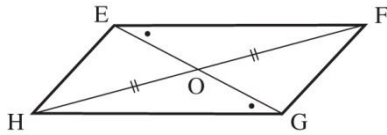
- (a) False (It could also be a rhombus.)
 (b) False



Prove that MNST is a parallelogram.

$\angle MTS + \angle TSN = 180^\circ$
 \therefore Co-interior angles are supplementary.
 $\therefore MT \parallel NS$.
 \therefore MNST is a parallelogram since both pairs of opposite sides are parallel.

28



$HO = OF$ and $\angle HGO = \angle OEF$.
Prove that EFGH is a parallelogram.

$EF \parallel HG$ (Since alternate angles are equal,
 $\angle HGO = \angle OEF$.)

Also

$HO = OF$ (given)

$\angle HGO = \angle OEF$ (Alternate angles are equal.)

$\angle EOF = \angle GOH$ (Vertically opposite angles are equal.)

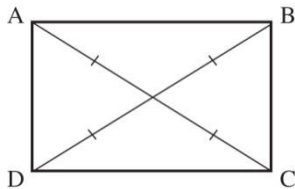
$\therefore \triangle EOF \cong \triangle GOH$ (ASA)

$\therefore EF = GH$ (Corresponding sides in congruent triangles are equal.)

$\therefore EF$ and HG are equal and parallel.

$\therefore EFGH$ is a parallelogram.

29

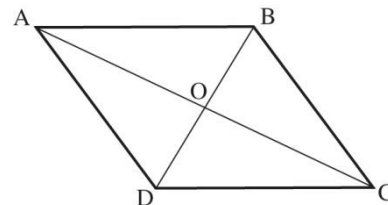
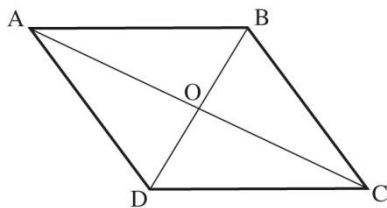


ABCD is a rectangle since the diagonals AC and BD are equal and bisect each other.

What sort of quadrilateral is ABCD?

30 In a quadrilateral, $AB = CD$, $DO = OB$ and $AC \perp BD$.

Show that ABCD is a rhombus.



$AB = CD$ (given)

$\angle AOB = \angle COD$ (given 90°)

$DO = OB$ (given)

$\therefore \triangle AOB \cong \triangle COD$ (RHS)

$\therefore AO = CO$ (Corresponding sides in congruent triangles are equal.)

\therefore Diagonals bisect each other at right angles.

$\therefore ABCD$ is a rhombus.

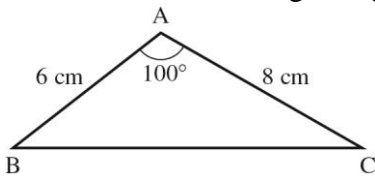
Jacaranda Maths Quest 10 + 10A: Measurement and geometry: Geometric reasoning

Topic 10: Deductive geometry
Test A

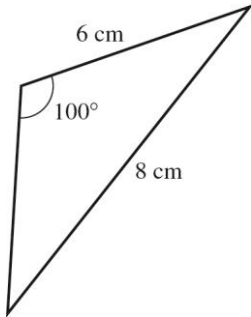
Name: _____

FLUENCY

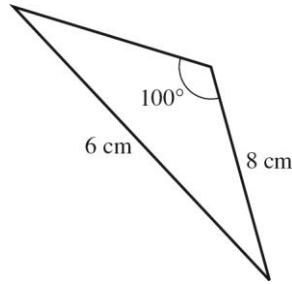
1 Which of the following triangles would be congruent to $\triangle ABC$?



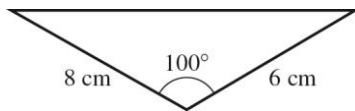
A



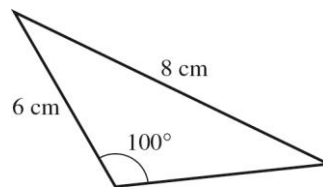
B



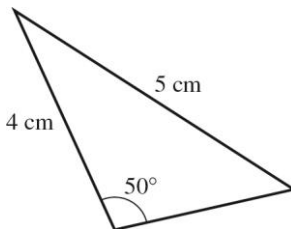
C



D



E



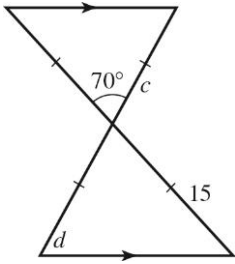
Mark

1

Progression point

8.0

2 The values of c and d , respectively, in the pair of congruent triangles in the figure below are:

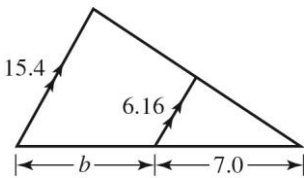


- A 15 and 55°
- B 7.5 and 25°
- C 15 and 70°
- D 15 and 110°
- E 7.5 and 55°

1

8.0

3 The value of b in the figure below is:

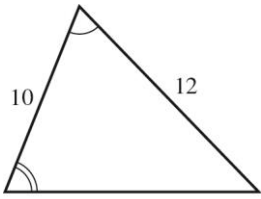


- A 6.16
- B 7.0
- C 10.5
- D 15.4
- E 13.16

1

9.5

4 The value of a in the pair of similar triangles below is:

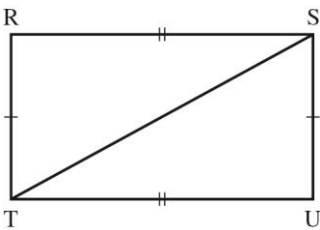


- A 2
- B 3
- C $3\frac{1}{8}$
- D $6\frac{1}{4}$
- E $3\frac{1}{4}$

1

9.25

5 The test that could be used to prove that $\triangle TRS \cong \triangle SUT$ is:



- A SAS
- B AAS
- C AAA
- D RHS
- E SSS

1

9.75

6 $\triangle ABC \cong \triangle RST$. This means $\angle ACB$ equals:

- A $\angle RST$
- B $\angle RTS$
- C $\angle SRT$
- D $\angle TRS$
- E $\angle AST$

1

8.0

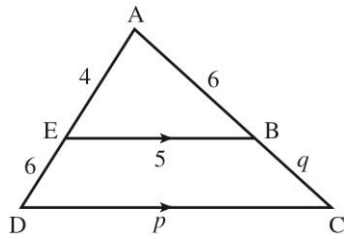
<p>7 Which of the following properties are not true about a parallelogram?</p> <p>A Opposite sides are parallel. B Opposite sides are equal. C Diagonals are equal. D Opposite angles are equal. E Angles sum to 360°.</p>	<p>1</p>	<p>8.5</p>
<p>8 A trapezium is:</p> <p>A a parallelogram with one right angle B a quadrilateral with both pairs of opposite sides parallel C a rectangle with two adjacent sides equal D a quadrilateral with at least one pair of opposite sides parallel E a quadrilateral with two sides that are equal</p>	<p>1</p>	<p>8.5</p>
<p>9 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is best described as:</p> <p>A a trapezium B a kite C a rectangle D a parallelogram E a rhombus</p>	<p>1</p>	<p>8.5</p>
<p>10 True or false?</p> <p>(a) A parallelogram is a square. (b) A square is a parallelogram.</p>	<p>2</p>	<p>8.5</p>

UNDERSTANDING

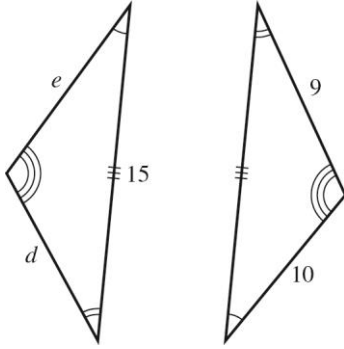
11 Find the values of p and q in the diagram below.

6

9.5



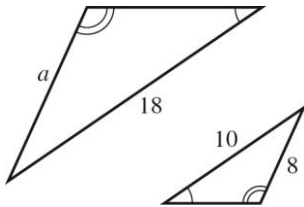
12 Find the values of d and e in the following pair of congruent triangles.



2

8.0

13 Find the value of a in the pair of similar triangles.



2

9.25

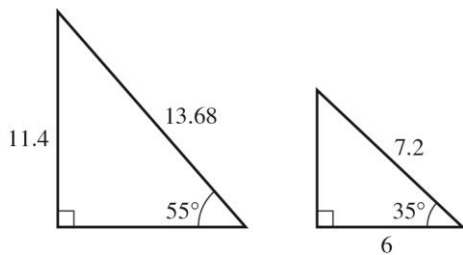
14 Name any quadrilaterals whose diagonals bisect the angles they pass through.

2

8.75

15 Test whether the following triangles are similar. For similar triangles, find the scale factor.

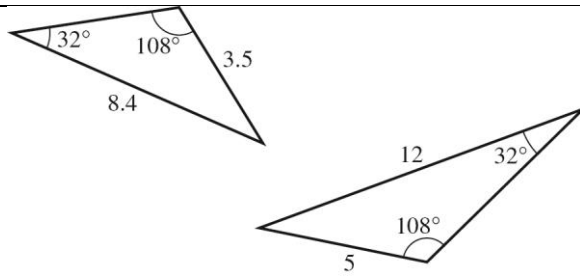
(a)



(b)

2

9.25

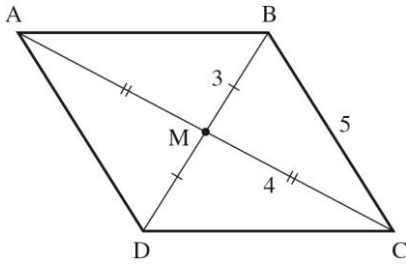


REASONING

16 Prove that ABCD is a rhombus.

3

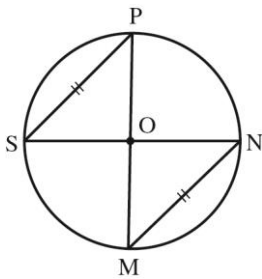
10.0



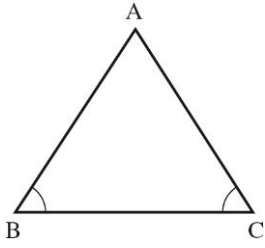
17 O is the centre of the circle.
Prove that $\Delta SPO \cong \Delta MNO$.

3

10.0



- 18** Prove that if two angles of a triangle are equal, then the sides opposite those angles are also equal.

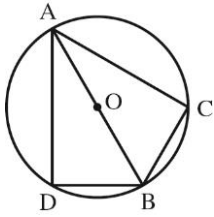


4

10.0

PROBLEM SOLVING

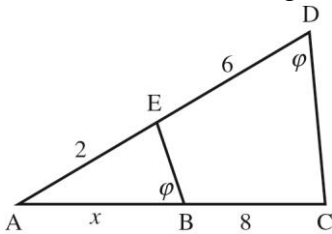
- 19** AB is a diameter of a circle. If $BC = BD$, prove that triangle ABC is congruent to triangle ABD.



3

10.0

- 20** Find x in the following diagram.



4

9.75

Total marks: 42

